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RECONSTRUCTABILITY ANALYSIS OF MULTI-DIMENSIONAL RELATIONS: A Theoretical Basis for Computer-Aided Determination of Acceptable Systems Models†

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Reconstructability analysis is viewed as a process of investigating the possibilities of reconstructing desirable properties of overall systems from the knowledge of the corresponding properties of their various subsystems. Overall systems are represented by n-dimensional relations, each of which is viewed as a finite set of aggregate states (n-tuples) formed by individual states of n variables. Depending on the context of investigation, the relations may include various measures defined on them, such as probability distributions or fuzzy set membership functions. Each subsystem is defined as a projection of the overall relation into k of its n dimensions ($k<n$). Every set of subsystems of an overall system is referred to as a structure system and is viewed as a possible basis for reconstructing specified properties of the overall system.

This paper focuses on the problem of generating, in an orderly fashion, all meaningful structure systems of a given overall system which are desirable in each particular situation. Some classes of structure systems, each of which emerges either from computational considerations or from various investigative contexts, are introduced and examined. The ultimate result of the paper is a set of eight schematically described generative procedures which provide a comprehensive basis for developing two computationally efficient software packages for reconstructability analysis, one for interactive processing and one for batch processing.

INDEX TERMS: Reconstructability analysis, multi-dimensional relations, systems modelling, structure system, lattice of structure systems, structure identification, subsystems, structure refinement, structure refining procedure, structure aggregating procedure.

1 GENERAL DISCUSSION

As science's task becomes more and more accepted to be not merely "the simple description of nature" but, rather, something which emerges from the "interplay between ourselves and nature" (Heisenberg),[2,3] the characterization of forms which this interplay takes has become more and more recognized as a crucial area of study. We consider this interplay to be roughly synonymous with the term "modelling activity." The characterization and development of those aspects of this activity which are useful in conjunction with content supplied from any specific system is, in our opinion, one of the main tasks of general systems research.

In our perception, complex systems are such that this characterization must capture in a fundamental way the idea of the *process* of modelling rather than the construction of *a* model and we agree with Lofgren[30] that this process must embody the notion of continuous interplay between description and interpretation. We also feel that this characterization, to be useful, must include or supply operational and working algorithms which an investigator can use to be put in touch with specific systems of interest. The conception and elaboration of the interactive framework, *General Systems Problem Solver* (GSPS), accomplishes this goal for a wide spectrum of fundamental systems problems.[13,14]

One of the major classes of systems problems which GSPS addresses is that dealing with key issues which are associated with, for example, systems decomposition, simplification, and design, and which generally fit under the heading of structure modelling or structure determination. For this class of problems we are thus primarily concerned with the notion of structure.

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Webster defines **structure** as: *the interrelation of parts as dominated by the general character of the whole.* While this definition and other similar ones adequately convey a conceptual significance, they do not provide much guidance with respect to implementation and use in systems investigations. Consequently, it must be a high priority task of general systems research to clearly delineate issues involved in the identification or determination of structure, and to do this in a manner which supplies operational significance. The first step in this effort is to accept the widely recognized fact that virtually any communicable conception of system involves (if only implicitly) a set of variables[1,2] along with, for each variable, a set of states or values that are recognizable for that variable.

A useful way to consider the structure of a system is as a specification of a set of subsystems and connections between them along with a suitable "composing rule" such that some criteria defined in terms of the overall system or in terms of the structure itself are met. In terms of this conception, it is appropriate to consider a given structure as an hypothesis regarding the overall system. The individual subsystems may be considered as generating their own combinations of states, either independently of other subsystems or constrained by those to which they are connected or coupled. The "reasonableness" of the hypothesis then depends on the extent to which the desirable features of the overall system can be reconstructed from the hypothesis.

Structure determination of the form just described generally fits into one of two categories depending on whether or not so-called internal variables are allowed. Although we do not deal in this paper with situations which allow the use of internal variables, we offer a few comments regarding such cases for purposes of comparison. Situations in which internal variables are allowed arise most often in problems of design and are also evidenced in some modelling approaches. If there is no restriction on the use of internal variables, the designer or modeller has much more freedom and, consequently, such cases admit much more easily of the derivation of acceptable structures. We can draw a parallel here between the work of a designer—who has available for use in construction or design a set or store of available modules (whether these are actual or conceptual)—and an investigator trying to develop a "theory" regarding some object of investigation. In this latter case, the modules may be previously completed theoretical or empirical studies which overlap the domain of interest of the investigator. In this case, however, the use of these modules imposes significant constraint on the investigator. Moreover, it does not seem to be conceptually significant to allow the theory-builder general freedom in the use of internal variables as their use would then entail essentially no external reference. For these reasons, studies relating to the use of internal variables are quite tricky at the general level since there is no semantics which naturally accrues to them. We thus conceive that the use of internal variables implicitly defines a new system including these variables in the original configuration, and we restrict our attention to structure determination which is always based on a fixed set of variables.

One of the most meaningful criteria to use in the evaluation of various structures (hypotheses) is the extent to which the constraint which the overall system exhibits can be reconstructed from consideration only of constraints exhibited by subsystems of the structure. In terms of variables and their states, subsystems are represented by subsets of the set of variables associated with the overall system; constraint in the overall system as well as in individual subsystems is expressed in terms of the respective sets of variables and their states. In this paper we consider overall systems as n-dimensional relations, where n is the number of variables associated with the system. Each such relation is viewed as a finite set of aggregate states (n-tuples) formed by states of the individual variables. Depending on the context of particular investigations the set of states may or may not include a measure defined on it, such as a probability distribution or a fuzzy set membership function. Furthermore, the relation may involve lagged variables, may describe transitions between states, etc. No special properties of the variables, such as a metric or ordering relation, are required for the analysis described here.

The term "*reconstructability problem*" is usually used to refer to the problem of determining such subsystems of an overall system that the constraint of the overall system can be adequately constructed from the constraints associated with these subsystems.

Consider, as an example of the reconstructability problem, that hardware monitoring has been performed for four variables on a computer

system to find conditions under which the utilization of CPU (central processing unit) is high. Suppose that the variables represent the utilization of CPU and three channels, say channel 1 (C1), channel 2 (C2) and channel 3 (C3). Suppose further that the monitoring was performed in the period of one hour of a typical workload and in each interval of 10 seconds the utilization of each of the observed units was recorded. If it was smaller than a certain threshold defined by the investigator, it was considered low (L) during the interval; if it was higher than the threshold, it was considered high (H). Assume that the following states were observed with approximately the same frequency.

CPU	C1	C2	C3
H	L	H	H
H	H	L	L
L	L	L	L
L	H	H	H
H	L	H	L
L	L	L	H

This experimental result indicates that the variables are strongly related: the four-dimensional (quarternary) relation contains only 6 states (4-tuples) out of 16 potential states. It provides the investigator with an important insight: to keep the utilization of CPU high, make arrangements under which the utilization of the three channels is mixed in one of the following three ways.

C1	C2	C3
L	H	H
H	L	L
L	H	L

Observe that when considering only binary relations based on the individual pairs of variables (which are two-dimensional projections of the overall quarternary relation), we obtain

CPU	C1
H	L
H	H
L	L
L	H

CPU	C2
H	H
H	L
L	L
L	H

CPU	C3
H	H
H	L
L	L
L	H

C1	C2
L	H
H	L
L	L
H	H

C1	C3
L	H
H	L
L	L
H	H

C2	C3
H	H
L	L
H	L
L	H

Each of these relations contains all potential two-dimensional aggregate states. This indicates that the variables are pair-wise independent. Yet, they are far from being independent when considered all together. Consequently, no data based on the observation of pairs of variables would provide a basis for constructing the overall quarternary relation in this case; the overall relation is simply not reconstructable from its two-dimensional projections.

However, it is not the case that the overall relation is not reconstructable from any lower (than four) dimensional relations. In this case the relation is reconstructable from the following two three-dimensional relations R_1 and R_2 (that is, two coupled subsystems, each with three variables).

$R_1 =$

CPU	C1	C2
H	L	H
H	H	L
L	L	L
L	H	H

$R_2 =$

C1	C2	C3
L	H	H
H	L	L
L	L	L
H	H	H
L	H	L
L	L	H

Even though these two relations may have been obtained from two different hardware monitors, they obviously are equal to relations obtained by taking projections of the overall relation into the dimensions associated with the two subsystems. This structure can be illustrated by the block diagram in Figure 1a.

For purposes of testing reconstructability, we observe that the two subsystems are coupled or joined by variables C1, C2 and the operation of *relational join* is directly applicable. In general, the relational join is defined for two relations which overlap in some of their dimensions and gives as a result an overall relation which is a conjunction of the two and which takes into account each of the two relations. Formally, the join $P * Q$ of two binary relations $P \subset A \times B$ and $Q \subset B \times C$ is a ternary relation defined as $P * Q$

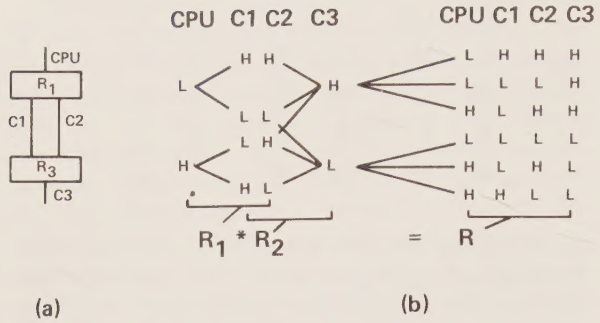

FIGURE 1 Illustration of the relational join.

$=\{(a,b,c)|(a,b)\in P$ and $(b,c)\in Q\}$. In the discussed example, sets A, B, C represent, respectively, states of variable CPU, aggregate states of variables C1 and C2, and states of variable C3 (the order can also be inverted); the use of the relational join in this example is depicted in Figure 1b. We can see that the join of these two subsystems does in fact give us the overall system exactly.

It is worth observing that the block diagram in Figure 1a, which has been determined to completely capture the total constraint of the overall system, indicates that C3 is decoupled from CPU. This is reflected by the fact—which can be seen in the original four-dimensional relation—that the statement about high CPU utilization can be made solely in terms of C1 and C2: to keep the utilization of CPU high make arrangements under which the utilization of either C1 or C2 is high while the utilization of the others is kept low.

This simple example illustrates one of the most basic principles of systems thinking: *Do not a priori assume that a whole system can be reconstructed from the full knowledge of its parts; even if it is reconstructable, do not assume that it is reconstructable from the full knowledge of an arbitrarily chosen collection of parts.*

Although the importance of justifying reconstructability properties in the various areas of systems modelling is rather obvious, this problem has been largely neglected by systems researchers. A rare exception was the late Ross Ashby who suggested the concept of *cylindrance* for multidimensional relations and developed an algorithm through which it is possible to determine whether a given relation of dimension n can be reconstructed from its projections of a particular dimension $k<n$.[3,4,31] The reconstruction, if possible, is accomplished by the set intersection of extensions (cylinders) of all k-dimensional projections of the given relation.

Ashby's work is fundamental. However, there is no need to only consider, as potential hypotheses, structures where each subsystem involves the same number of variables; nor is it necessary, in any given structure, to include all of the subsystems with the number of variables under consideration. In the general reconstructability problem, then, any collection of subsystems of any number of variables is a potential structure hypothesis. We define **reconstructability analysis** to be the *process of investigating the possibilities of reconstructing desirable properties of overall systems from the knowledge of the corresponding properties of their various subsystems.* Some initial results regarding the reconstructability analysis were presented by the authors at the Fourth European Meeting on Cybernetics and Systems Research at Linz, Austria, in March 1978.[15] While meaningful classes of the set of all structures were introduced in the Linz presentation with the objective of forming a comprehensive methodological basis for reconstructability analysis, this paper elaborates on these results to the point of the specification of well-defined and computer-implementable procedural schemes for structure hypotheses formation.

2 SYSTEMS AND THEIR SUBSYSTEMS

To give a precise formulation of the general reconstructability problem, we consider a system to be represented by a finite set of *variables* $V=\{v_1, v_2, \ldots, v_n\}$, each associated with a *state set* in the family $\mathscr{V}=\{V_1, V_2, \ldots, V_m\}$ by function (onto and, generally, many-to-one):

$$s: V \to \mathscr{V};$$

$s(v_i)$ is viewed as the state set of variable v_i. We call an element of the Cartesian product

$$A = s(v_1) \times s(v_2) \times \ldots \times s(v_n)$$

an *aggregate state* of the system. We do not make any statements as to the nature of the variables allowing, for example, that each be directly observable, or that some represent combinations of observable variables, or that a given variable may represent past or future states of another variable, etc. While such distinctions are

important for an overall investigation, they are not relevant to the particular problem we discuss here. We assume that each system also involves either a specification of a subset of the aggregate states or an assignment of some measure, such as a probability or a fuzzy set membership grade, to each of the aggregate states; in the first case we define a system merely by specifying which of its aggregate states occur. Thus, the meaning of the term "overall system" as used in this paper involves a mapping b, frequently referred to as behavior,[14,24] from the set of all aggregate states (associated with the variables in set V) to some other set, say set Q:

$$b: A \to Q.$$

In the case where we deal only with occurrence or non-occurrence of aggregate states, $Q=\{0,1\}$ with the interpretation that states which occur in the system are mapped to 1. In the case, for example, where we are dealing with a probability distribution, Q would be the interval $[0,1]$ of real numbers from 0 to 1, inclusive. The specification of the function b may either result from data gathering and processing or (as in the case of design problems) may be given directly by a "client."

We summarize these considerations, then, by defining an *overall system*, B, as the six tuple

$$B=(V, \mathscr{V}, s, A, Q, b),$$

where:

- $V=\{v_i | i \in N_n\}$ $(N_n=\{1,2,\ldots,n\})$;
- $\mathscr{V}=\{V_j | j \in N_m\}$ $(N_m=\{1,2,\ldots,m\},\ m \leqq n)$;
- $s: V \to \mathscr{V}$;
- $A = \underset{i \in N_n}{\times} s(v_i)$;
- Q is a set representing a measure;
- $b: A \to Q$.

Given an overall system, say B, a *subsystem* of it, say B', is defined in a similar manner but is based on some subset V' of the original set V of variables and all of the entities involved in B' depend in some way on the corresponding entities in B. Let

$$B'=(V', \mathscr{V}', s', A', Q', b')$$

be a subsystem of B. Then, the following conditions must be satisfied:

- $V'=\{v_i | v_i \in V \text{ and } i \in Z, \text{ where } Z \subset N_n\}$;
- $\mathscr{V}' \subseteq \mathscr{V}$ such that s' is onto;
- function $s': V' \to \mathscr{V}'$ is defined in the same way as function s but for the restricted domain V' and with such range which makes s' onto;
- $A' = \underset{i \in Z}{\times} s'(v_i)$;
- $Q'=Q$;
- $b': A' \to Q$ such that $b'(\alpha)=f(\{b(\beta) | \beta \in z^{-1}(\alpha)\})$, where:
  - (i) $z: A \to A'$ such that $z(x)=z(y)$ iff x and y are equal in states of all variables in V' (identified by $Z \subset N_n$);
  - (ii) f is determined by the nature of function b and, possibly, by some context dependent criteria.

We describe three important special cases which illustrate the meaning of f and then give an example which illustrates all of the introduced concepts:

(a) If $Q=\{0,1\}$, then f is the MAX function and

$$b'(\alpha)=\underset{\beta}{\text{MAX}}\{b(\beta)\} \quad \text{for all } \beta \in z^{-1}(\alpha);$$

(b) If Q is $[0,1]$ and $b(x)$ is the probability of x, then f is the summation function and

$$b'(\alpha)=\sum_{\beta} b(\beta) \quad \text{for all } \beta \in z^{-1}(\alpha)$$

(c) If Q is $[0,1]$ and $b(x)$ is a fuzzy membership grade of x, then f may be the MAX function ($b'(\alpha)$ is the same as in (*a*)) but other functions can also be used depending on the context of the investigation (e.g., AVERAGE, SUM, etc.).

Example For an overall system B, let $V=\{v_1, v_2, v_3\}$, $\mathscr{V}=\{V_1, V_2\}$, and $s(v_1)=s(v_2)=V_1$, $s(v_3)=V_2$, where $V_1=\{0,1\}$ and $V_2=\{0,1,2\}$; then, $A=V_1 \times V_1 \times V_2$ and $N_3=\{1,2,3\}$. Consider two alternatives of Q and b using the same sets V and A: $Q_1=\{0,1\}$, $Q_2=[0,1]$, where b_1 and b_2, defined by Table I, are a selection function and a probability distribution function on A, respectively. For a subsystem B', let $V'=\{v_1, v_2\}$ (i.e., $Z=\{1,2\}$). Table II illustrates the function z relevant to this subsystem. We now specify b'_1 and b'_2, for subsystems using Q_1 and Q_2, respectively, by Table III, which is constructed from the information in Tables I and II.

TABLE I

$x \in A$				
v_1	v_2	v_3	$b_1(x)$	$b_2(x)$
0	0	0	1	0.2
0	0	1	0	0.0
0	0	2	1	0.3
0	1	0	0	0.0
0	1	1	0	0.0
0	1	2	0	0.0
1	0	0	1	0.03
1	0	1	1	0.07
1	0	2	1	0.1
1	1	0	0	0.0
1	1	1	0	0.0
1	1	2	1	0.3

TABLE II

	$x \in A$			$z(x) \in A'$	
	v_1	v_2	v_3	v_1	v_2
$\{z^{-1}(00)\} =$	0	0	0	0	0
	0	0	1	0	0
	0	0	2	0	0
$\{z^{-1}(01)\} =$	0	1	0	0	1
	0	1	1	0	1
	0	1	2	0	1
$\{z^{-1}(10)\} =$	1	0	0	1	0
	1	0	1	1	0
	1	0	2	1	0
$\{z^{-1}(11)\} =$	1	1	0	1	1
	1	1	1	1	1
	1	1	2	1	1

TABLE III

$\alpha \in A'$		$b_1'(\alpha) =$	$b_2'(\alpha) =$
v_1	v_2	$\underset{x}{\text{MAX}}\{b_1(x)\}$ for all $x \in z^{-1}(\alpha)$	$\underset{x}{\sum} b_2(x)$ for all $x \in z^{-1}(\alpha)$
0	0	MAX $\{1, 0, 1\} = 1$	$0.2 + 0 + 0.3 = 0.5$
0	1	MAX $\{0, 0, 0\} = 0$	$0 + 0 + 0 = 0$
1	0	MAX $\{1, 1, 1\} = 1$	$0.03 + 0.07 + 0.1 = 0.2$
1	1	MAX $\{0, 0, 1\} = 1$	$0 + 0 + 0.3 = 0.3$

While the definitions as given may seem unnecessarily elaborate, all aspects are necessary for the **general reconstructability problem**: *to identify from among all possible collections of subsystems those which allow the reconstruction of desirable properties of the overall system under given quality criteria.*

It follows from the definition of subsystem that, given an overall system and a specifi context of investigation through which function is chosen, the set of subsets of its variables is i one-to-one correspondence with the set of all it subsystems. That is to say, given an overal system and a specific formulation of the recon structability problem, each subset of variable can be used to uniquely identify a subsystem Consequently, it is advantageous to work onl with subsets of variables as subsystem identifier during all stages of the reconstructability analysi except those where the full description i unavoidable.

3 CLASSES OF STRUCTURES

Let V be a set of variables which represents system under investigation. As described i Section 2, subsets of V can, for some purpose uniquely identify subsystems of the overall sys tem. For the purpose of the complete characteriz ation and classification of the set of all possibl collections of subsystems of a given system, w take advantage of this simple identification o subsystems by subsets of variables. Using thi convention we define a *structure* to be any fami of subsets of V and we can define the set of a possible structures with respect to V as

$$S = \{S_i | S_i \subseteqq \mathscr{P}(V)\}$$

where $\mathscr{P}(V)$ denotes the power set of V. Althoug the symbol "S" does not indicate that the set o structures depends on the set V, we take it as convention in this paper that if the distinction not made, the symbols which we use are alway understood to refer to the same set V.

In what follows in this section we define an characterize some important subsets of S. Th significance of each of them emerges either fro computational considerations or from variou contexts of investigation, in which only som more specific types of structures are applicable o desirable.

As a basis for introducing appropriate subset of S and of deriving simple computationa schemes, let us define a mapping

$$r: S \to \hat{R},$$

where $\hat{R}$ is the set of all symmetric binary rela

tions defined on set V (undirected graphs), and $r(S_i)$ is the binary relation in which variables v_j and v_k are related if and only if they both belong to at least one of the subsets of V through which S_i is defined. Formally,

$$r(S_i)=\{(v_j, v_k)|v_j, v_k \in V, \exists(E_a \in S_i)(v_j \in E_a, v_k \in E_a)\}.$$

We shall refer to elements of set $\hat{R}$ as *graphs*. Although this is compatible with standard terminology,[9] it should be recalled that the graphs in set $\hat{R}$ are symmetric (undirected). Examples illustrating mapping r are shown in Figure 2, where variables are identified by integers. In depicting elements of $\hat{R}$, we do not bother in this paper to draw loops on the nodes.

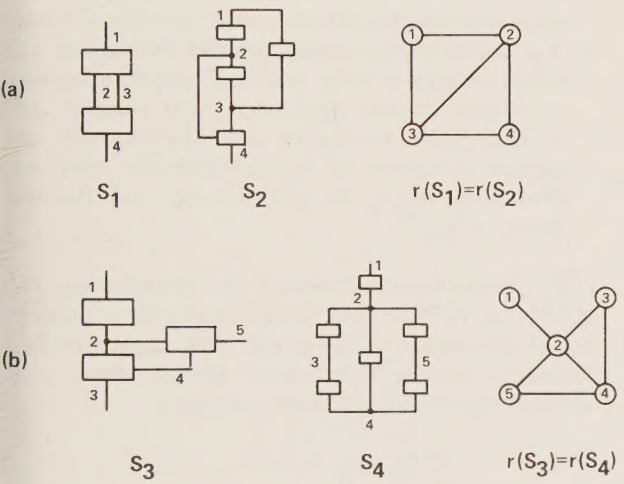

FIGURE 2 Examples illustrating mapping r.

Mapping r is clearly many-to-one and onto. As such, it imposes the following equivalence relation on the set S of all structures:

$$S_i \overset{r}{\equiv} S_j \quad \text{if} \quad r(S_i)=r(S_j).$$

We shall use the symbol S/r to denote the set of equivalence classes imposed on S by r. Clearly, there is a natural one-to-one correspondence between equivalence classes in S/r and graphs in $\hat{R}$.

Set $\hat{R}$ together with the relation of "being a subset of" (or, alternatively, the operations of set union and intersection) form a Boolean lattice. The one-to-one correspondence between S/r and $\hat{R}$ imposes then an isomorphic Boolean lattice on the set S/r.

For each of the subsets of S that are introduced in this paper, a mapping similar to r is defined but with its domain restricted to the respective subset of S which is under consideration; such mappings are denoted by r with subscripts identifying the considered domains.

Some structures S_i in set S may not preserve the original conception of the object of investigation (if S_i does not constitute a cover of V). Such structures are not meaningful in the reconstructability problem. The requirement of covering set V leads to the definition of the following subset of S:†

$$S_\alpha=\{\alpha_i|\alpha_i \subset \mathscr{P}(V), \bigcup_{E_a \in \alpha_i} E_a = V\}.$$

Some structures α_i contain the empty set $\varnothing$ which is not meaningfull in the reconstructability problem. If we exclude them from set S_α, we obtain a set S_H of structures which are also known as hypergraphs:[9]

$$S_H=\{H_i|H_i \subset (\mathscr{P}(V)-\varnothing), \bigcup_{E_a \in H_i} E_a = V\}.$$

The level of generality of hypergraphs is still higher than necessary for the purposes of the reconstructability analysis. A structure H_i may contain subsystems which contribute no information which is not present in some other subsystems of H_i. This occurs in most application areas when there exist two subsystems $E_a, E_b \in H_i$ such that $E_a \subset E_b$. We thus further restrict the set of possible structures by defining

$$S_G=\{G_i|G_i \subset \mathscr{P}(V), \bigcup_{E_a \in G_i} E_a = V, E_a \not\subset E_b$$

$$\text{for each pair} \quad E_a, E_b \in G_i (a \neq b)\}.\ddagger$$

Every structure in S_G is characterized by two properties: (i) each variable of the overall system participates in at least one of the subsystems; (ii) no subsystem contains only variables which are included in any other subsystem of the same structure.

It follows from the previous discussion that structures which are not included in the set S_G may be disregarded as hypotheses in the reconstructability analysis without any loss of gene-

†The symbol α as well as some other symbols in this paper are used to retain a connection with previous work.[24-28]

‡Observe that the empty set is excluded by the requirement $E_a \not\subset E_b$.

rality. However, none of the structures in S_G can be disregarded as an hypothesis in the general formulation of the reconstructability problem. Hence, it is reasonable to refer to structures in set S_G as *general structures* or *G-structures.*

In terms of the reconstructability problem, our ultimate goal is a computationally efficient procedural scheme which will allow consideration and evaluation of all general structures for a given set of variables. As shown in Section 6, this goal can be accomplished by utilizing appropriate subsets of S_G as well as a natural partial ordering which can be defined on the set S_G in the following way.

Let G_i and G_j be elements of S_G. Then $G_i \leqq G_j$ if and only if for each set $E_a \in G_i$ there exists a set $E_b \in G_j$ such that $E_a \subseteq E_b$. If $G_i \leqq G_j$, we say that "structure G_i is a *refinement* of G_j" or, alternatively, that "G_j is an *aggregate* of G_i." Moreover, we say that "G_i is an *immediate refinement* of G_j" if and only if $G_i \leqq G_j$ and there is no G_k such that $G_i \leqq G_k$ and $G_k \leqq G_j$; similarly for the term "*immediate aggregate.*" Given a general structure G_i, we define its *structure neighborhood* as the set of all immediate refinements and all immediate aggregates of G_i. We observe that the set S_G together with the defined partial ordering also constitute a lattice. This can be easily seen by observing that: (i) there exists a universal upper bound—set V; (ii) there exists a universal lower bound—set $\{\{v_1\}, \{v_2\}, \ldots, \{v_n\}\}$; (iii) for each pair $G_i, G_j \in S_G$, the greatest common refinement $\mathrm{REF}(G_i, G_j)$ is defined by $\mathrm{REF}(G_i, G_j) = \{E_a | E_a \in K_{i,j}$ and $E_a \not\subseteq E_b$ for each pair $E_a, E_b \in K_{i,j}\}$, where $K_{i,j} = \{E_p \cap E_q | E_p \in G_i, E_q \in G_j\}$; (iv) for each pair $G_i, G_j \in S_G$, the least common aggregate $\mathrm{AGG}(G_i, G_j)$ is defined by $\mathrm{AGG}(G_i, G_j) = \{E_a | E_a \in L_{i,j}$ and $E_a \not\subseteq E_b$ for each pair $E_a, E_b \in L_{i,j}\}$, where $L_{i,j} = G_i \cup G_j$.

We define now the mapping

$$r_G: S_G \to R$$

in the same way as mapping r but with R being the set of all symmetric *and reflexive* binary relations defined on set V (undirected graphs with loops, compatibility or tolerance relations). This mapping is clearly many-to-one and onto. Let the symbol S_G/r_G denote the set of equivalence classes imposed on S_G by r_G.

As mentioned previously for set S/r, there is also a Boolean lattice defined on set S_G/r_G which is isomorphic with the natural Boolean lattice defined on R. This isomorphism enables us to generate equivalence classes in S_G/r_G by appropriately operating on graphs in R (see Section 6). It is desirable, however, that each equivalence class in S_G/r_G be uniquely represented by some canonical structure. For that purpose, we introduce the following subsets of S_G:

- S_M, consisting of such general structures G_i which contain only maximal compatibility classes[33] associated with the binary relation $r_G(G_i)$ or, in another terminology, are based on cliques of graph $r_G(G_i)$.[9,22] We denote structures in S_M by M_i and refer to them as *M-structures.* Thus, a general structure G_i is also an M-structure if and only if for each $E_a \in G_i$ there is no $v_k \in V$ such that $v_k \notin E_a$ and $(v_j, v_k) \in r_G(G_i)$ for all $v_j \in E_a$. Let mapping $r_M: S_M \to R$ be defined in exactly the same way as mapping r but for the restricted domain S_M.
- S_C, consisting of such general structures G_i which contain only and *all* maximal compatibility classes associated with cliques of $r_G(G_i)$. Such structures are also known as complete covers of $r_G(G_i)$, and we refer to structures in S_C as *C-structures* and denote them by C_i.

An important motivation for introducing C-structures is that each graph contains a unique set of all cliques in the graph and, consequently, defines a unique C-structure. Hence, there is a natural one-to-one and onto mapping

$$r_C: S_C \to R,$$

defined in exactly the same way as mapping r but for the restricted domain S_C; this mapping is useful in proving the following theorem.

THEOREM 1 *Each equivalence class in S_G/r_G contains exactly one C-structure.*

Proof (i) Let an equivalence class $X \in S_G/r_G$ contain more than one C-structure, say C-structures a, b. Since r_C is one-to-one, $r_C(a) \neq r_C(b)$, and since r_C and r_G are each restrictions of r, $r_G(a) = r_C(a)$ and $r_G(b) = r_C(b)$. These facts imply that $r_G(a) \neq r_G(b)$. Hence, X is not an equivalence class. (ii) Let an equivalence class $Y \in S_G/r_G$ contain no C-structure. The facts that $|S_C| = |R|$ (r_C is one-to-one) and $|S_G/r_G| = |R|$ (by definition) imply that $|S_C| = |S_G/r_G|$. In order to satisfy this equality, there must be an equival-

ence class in S_G/r_G which contains at least two C-structures. But, as shown in (i), such equivalence class does not exist in S_G/r_G.

Theorem 1 guarantees that C-structures are qualified as canonical representatives of the equivalence classes in S_G/r_G. In each equivalence class in S_G/r_G, the C-structure is also the least refined G-structure in that equivalence class. The most refined G-structure in each equivalence class is clearly a structure which is built by exactly those pairs of variables which are related in the graph representing the equivalence class and by those single variables which are isolated in the graph. Let structures of the latter kind be called *primitive structures* or *P-structures* and let the class of primitive structures be denoted by S_P. Given a graph ρ_k, representing an equivalence class in S_G/r_G, the corresponding P-structure $P_k \in S_P$ is defined as follows: For all $v_i, v_j \in V$ $(v_i \neq v_j)$, $\{v_i, v_j\} \in P_k$ if and only if $(v_i, v_j) \in \rho_k$ and, for all $v_i \in V$, $\{v_i\} \in P_k$ if and only if there is no $v_j \in V$ $(v_i \neq v_j)$ such that $(v_i, v_j) \in \rho_k$.

C-structures and P-structures can be viewed as two *canonical representations* of the equivalence classes of G-structures. Each equivalence class represented by a graph ρ_k is polarized by the two canonical structures, say structures C_k and P_k: $C_k \geqq P_k$ and all other G-structures in the equivalence class are always in between C_k and P_k. The one-to-one correspondences between R and S_C, S_P, respectively, impose Boolean lattices on sets S_C and S_P, respectively, which are isomorphic with the natural Boolean lattice defined on R.

As an example, Figure 3 illustrates the lattice defined on S_G for 3 variables as well as the Boolean lattices (mutually isomorphic) defined on R, S_C, S_P and S_G/r_G. The variables are identified by integers; the G-structures are represented by block diagrams and are labelled by two indices, one identifying the equivalence class and one used for distinguishing G-structures in that class.

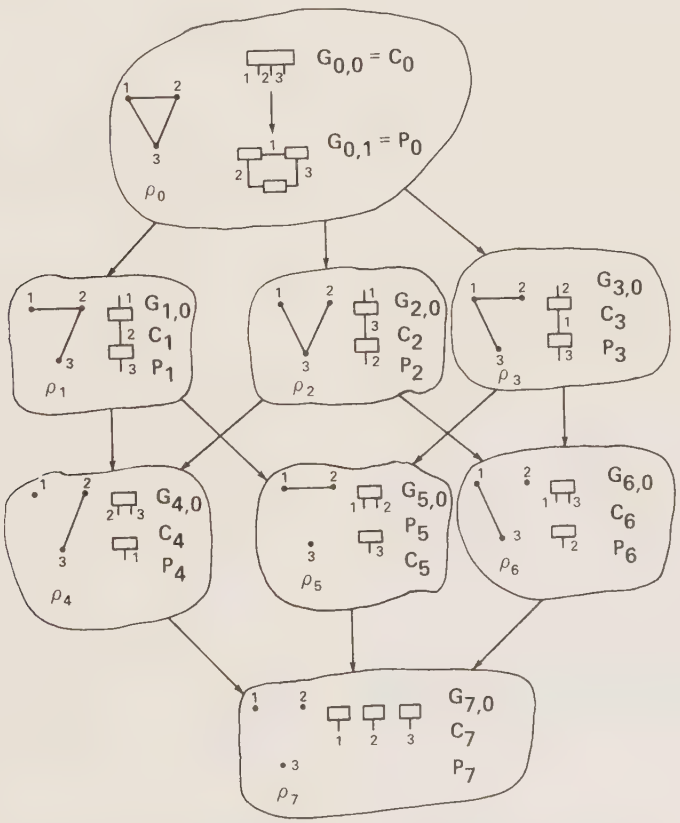

FIGURE 3 Lattice defined on S_G for 3 variables and Boolean lattices (isomorphic) defined on R, S_C, S_P and S_G/r_G for 3 variables.

It is useful to further introduce the following three subsets of S_M, each of which is desirable under some specific investigative contexts:

- S_I, consisting only of those M-structures which are irredundant covers of the associated graph $r_M(M_i)$. We refer to structures in S_I as *I-structures* and denote them by I_i. Thus, an M-structure M_i is also an I-structure if and only if $r_M(M_i-\{E_a\})\neq r_M(M_i)$ for each $E_a\in M_i$.
- S_β, consisting only of those M-structures M_i which satisfy the following condition: for each $E_a\in M_i$, the transitive closure of $r(M_i-\{E_a\})$ is a proper subset of the transitive closure of $r_M(M_i)$. We denote a structure in S_β by β_i, refer to it as a *β-structure*,† and observe that β-structures have the property that each subsystem of a β-structure contains information regarding at least one pair of variables which not only is not contained in any other subsystem but which is also not obtainable through any sequence of subsystems.

Consideration of β-structures is justified by the fact that it often makes sense to consider only structures where each subsystem contributes some unique information regarding the relationships which exist among the variables of the system. It will also prove useful to consider further restricted structures wherein if a pair of variables is directly related by participation in some subsystem, then that pair of variables is not indirectly related through a sequence of other distinct subsystems. The interpretive significance to this restriction stems from the consideration of a structure as an hypothesis regarding the contribution of each subsystem, and consequently of the interaction among variables, to the overall system. This restriction embodies the statement that if a hypothesis postulates a direct relation between two variables, then all interaction between the two variables, and all constraint in the overall system which is due to this interaction, are expressed through the direct relation. If such hypotheses are acceptable, this is of course a much stronger statement that can be made about the structure of a system. The last subset of S_M which we define, then, is:

- S_γ, consisting only of those M-structures defined in the following manner: Let $M_i\in S_M$; fo each pair $v_j, v_k\in V(v_j\neq v_k)$, let $E_{jk}=\{E_a|E_a\in M$ and $\{v_j,v_k\}\subseteq E_a\}$; then M_i is a γ-structure, γ_i, i and only if the pair (v_j,v_k) is not an element o the transitive closure of $r(M_i-E_{jk})$.†

†Extensive work has been done in reconstructability analysis limited to β-structures by Klir and Uyttenhove.[24-28,36]

4 SOME COMBINATORIAL CONSIDERATIONS

For combinatorial purposes, we can define a useful equivalence relation $\overset{i}{\equiv}$ on the set S of al structures in the following way: $S_j\overset{i}{\equiv}S_k$ if and only if S_j and S_k are isomorphic, i.e., one of them can be obtained from the other solely by a permutation (relabelling) of the variables.

Let S/i denote the set of equivalence classes o isomorphic structures (or permutation equivalence classes). Equivalence relation $\overset{i}{\equiv}$ can equally well be defined on any subset of S. Hence, S_G/i S_C/i, S_γ/i, etc., denote the sets of permutation equivalence classes of G-structures, C-structures γ-structures, etc., respectively. Moreover, $\overset{i}{\equiv}$ ca also be defined on the set R. Hence, R/i denote the set of permutation equivalence classes o graphs.

In order to avoid confusion, let $\overset{r}{\equiv}$ (introduced earlier) and $\overset{i}{\equiv}$ be referred to as *r-equivalence* an *i-equivalence*, respectively.

The i-equivalence is particularly useful in making catalogs of structures of certain kinds with a specific number of variables. It is obviously much easier to list only permutation equivalence classes rather than all structures of a desirable kind.

An example utilizing the i-equivalence is shown in Figure 4. Figure 4a represents the lattice shown in Figure 3 but simplified in the sense that isomorphic structures are not distinguished. This is done by deleting labels of variables in the block diagrams and including only one block diagram for each permutation equivalence class. Structures in each permutation equivalence class can easily be generated by permuting variables along entries of the blocks (subsystems). Symbols denoting the permutation equivalence classes (of G-, C-, and P-structures) are distinguished from symbols of the individual structures by asterisks in order to avoid ambiguity. The simplified lattice shown in Figure 4a is a homomorphic imag

†Examples of structures belonging to the various intersections of the sets defined in this section may be found in o previous paper.[15]

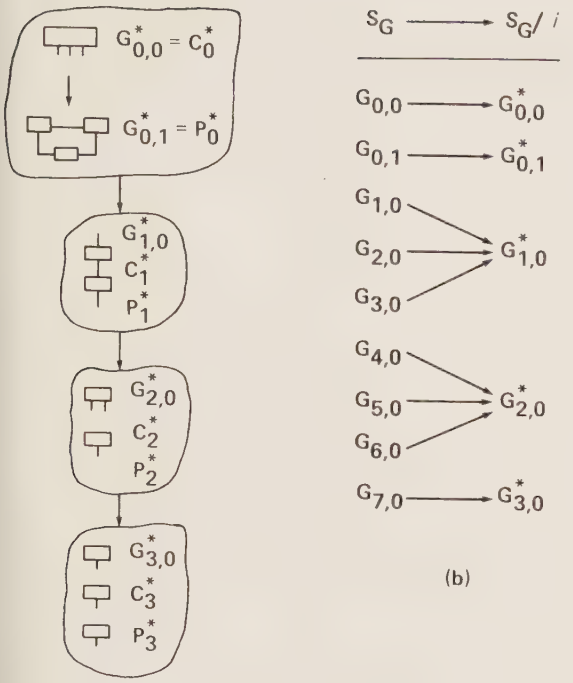

FIGURE 4 (a) Lattice defined on S_G/i for 3 variables (a simplified description of the lattice in Figure 3). (b) Homomorphic mapping $S_G \rightarrow S_G/i$ under which properties of the lattice in Figure 3 are preserved, in a simplified form, in the lattice in Figure 4a.

of the lattice in Figure 3. The homomorphic mapping, which is the basis for this simplification, is specified in Figure 4b.

The lattice defined on S_G/i for 4 variables is specified in Figure 5. Permutation equivalence classes are grouped together by the r-equivalence classes. Each of the latter classes is associated with a graph ρ^*_k and the two canonical structures, C^*_k and P^*_k. Permutation equivalence classes of G-structures are denoted by $G^*_{k,j}$, where k identifies the equivalence class in S_G/r_G and j makes distinctions within that class; by convention, $G^*_{k,0} = C^*_k$ and $G^*_{k,j_{\max}} = P^*_k$. To show more clearly the overall properties of this lattice, it is summarized in Figure 6, where the individual permutation equivalence classes of G-structures are denoted by identifiers k, j and the r-equivalence classes are more emphasized.

The lattice defined on S_C/i for 4 variables is described in Figure 7. Each permutation equivalence class is characterized by a block diagram without labels of its entries. The number placed next to each block diagram indicates the number of different C-structures in that permutation equivalence class; the number placed next to each arrow indicates the number of immediate refinements of each C-structure of one permutation equivalence class in the other class.

To get some feeling about the growth of structures in the various classes for growing number of variables, we summarize the known results of up to six variables ($n \leqq 6$) in Table IV; $|S|_n$, $|S_H|_n$, etc., stand for the numbers of structures for n variables in sets S, S_H, etc., respectively. Similar results regarding the permutation equivalence classes are given in Table V.

Results in those rows of Tables IV and V which are identified by asterisks are based on known analytic procedures and can, if desirable, be extended beyond $n=6$:

i) $|S|_n = 2^{2^n}$;

ii) $|S_H|_n = \dfrac{|S|_n - n|S|_{n-1}}{2}$ $(|S_H|_1 = 1)$;

iii) $|S_C|_n = |S_P|_n = 2^{n(n-1)/2}$ $(|S_C|_1 = |S_P|_1 = 1)$;

iv) a procedure for calculating $|S/i|_n$ was developed by Polya,[32]

v) $|S_H/i|_n = \dfrac{|S/i|_n - |S/i|_{n-1}}{2}$ $(|S_H/i|_1 = 1)$;

vi) the number $|S_C/i|_n = |S_P/i|_n$ is equal to the number of isomorphic classes of graphs with n nodes—the problem solved originally by Polya[32] (Harary[22] lists all isomorphic classes of graphs of up to $n=6$, Sloane[35] gives the number of classes up to $n=15$).

Results in those rows of Tables IV and V which are not identified by asterisks are experimental. Each of these rows represents an unsolved combinatorial problem. A particularly important and challenging problem is to find a procedure for calculating the numbers $|S_G|_n$ of general structures and the numbers $|S_G/i|_n$ of their permutation equivalence classes.

Complete catalogs of all i-equivalence classes of general structures for $n=3$, 4, 5 are given, respectively, in Tables VI, VII, VIII in Appendix A.

5 RELATIONSHIP AMONG CLASSES OF STRUCTURES

In considering further the rationale for the de-

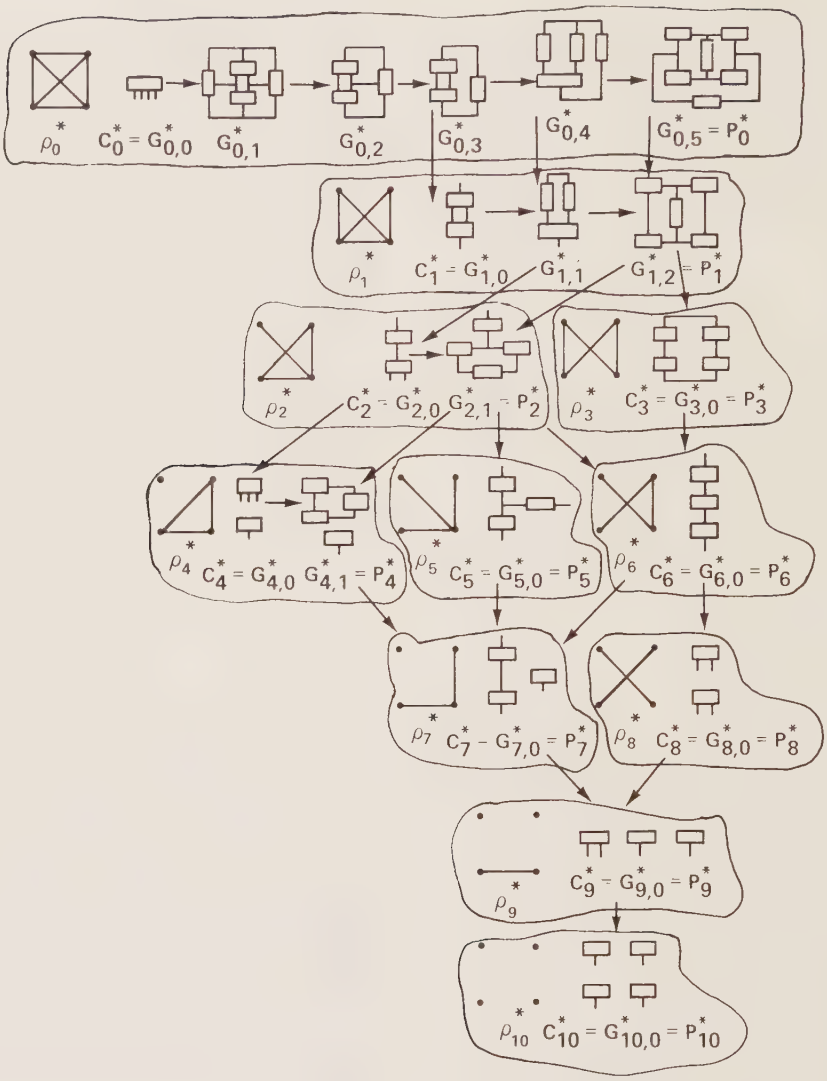

FIGURE 5 Lattice defined on S_G/i for 4 variables and lattices (isomorphic) defined on R/i, S_C/i, S_P/i, and $(S_G/r_G)/i$.

finitions in Section 3, we repeat that our goal is to delineate an overall procedure by which an investigator could generate—and thus conceivably "test"—every structural hypothesis regarding his object of investigation. The subset characterization which we have developed has been motivated by: (i) "meaningfulness," in context of possible interpretations of what subsystems may be; (ii) the fact that a particular characterization allows simplified computational procedures; (iii) some combination of the tw motivations. The definitions will thus provid direction for an operational procedure whos implementation will constitute a complete sof ware package to aid an investigator in th identification and determination of structures.

It is important to realize that the investigato is not required to be familiar with either th technical results or formal definitions of Sectio 3. All that is necessary for him or her to utili

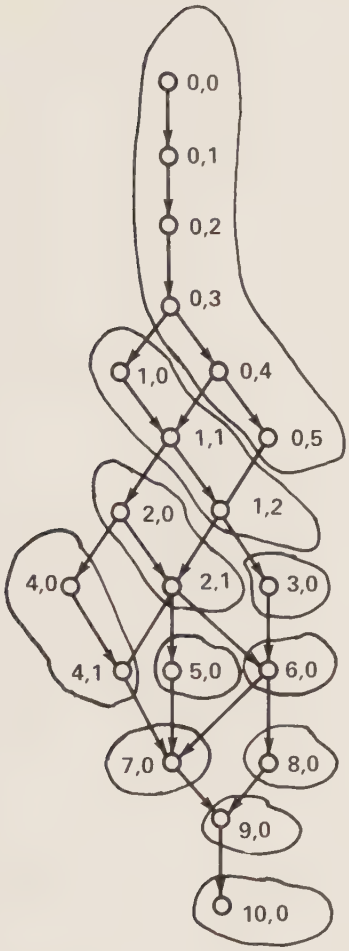

FIGURE 6 Summary of lattice defined on S_G/i for 4 variables (compare with Figure 5).

these developments is a general understanding of the distinctions which are represented by the definitions and a sense of how these characterizations relate to his own area of investigation and object language problem.[13,14]

The following are statements which, with the exception of statement (v), either follow immediately from the definitions or are easily shown:†

†Some of the statements do not hold for some low values of n; in particular, subset relations in the statements are all proper except for some low values of n. The term "low value of n" refers here to some value smaller than 6, depending on the statement under consideration. The "degenerate" cases for $n<6$ can easily be unravelled with the help of the available lists of graphs.[22]

i) $S_M \subset S_G \subset S_H \subset S_\alpha \subset S$;

ii) $S_C \subset S_M$;

iii) $S_\gamma \subset S_\beta \subset S_I \subset S_M$;

iv) $S_P \subset S_G$;

v) $S_\gamma \subset S_C$ (see Appendix B);

vi) r_M is not one-to-one and neither is any extension of r_M, i.e., r_G, r_H, r_α, r;

vii) r_C and r_P are one-to-one;

viii) r_C, r_P, r_I, r_M, r_G, r_H, r_α, r are all onto;

ix) r_β is not onto: to see this; consider graph ρ_3^* in Figure 5 for which no β-structure exists;

x) it follows from (iii) and (ix) that r_γ is not onto;

xi) it follows from (ix) that $S_C \not\subset S_\beta$; the fact that, e.g., the structure $\{\{v_1, v_2, v_3\}, \{v_3, v_4, v_5\}, \{v_5, v_6, v_1\}\}$ is a β-structure but not a C-structure shows that $S_\beta \not\subset S_C$; hence, S_β and S_C are non-comparable sets of structures with respect to the relation "being a subset of;"

xii) neither $S_C \subset S_I$ nor $S_I \subset S_C$, i.e., sets S_C and S_I are not comparable.

The listed properties imply that the considered classes of structures are partially ordered by the relation "being a subset of." This partial ordering, subject to the qualification that some classes merge for low values of n, is expressed in terms of the Hasse diagram in Figure 8. Classes identified by asterisks provide a basis for the computational procedures (Section 6); the other classes are motivated by the various investigative contexts.

6 PROCEDURES FOR GENERATING STRUCTURE HYPOTHESES

The kernel of the reconstructability analysis is the *generation of all desirable general structures*, each of which is viewed as a meaningful hypothesis representing a particular reconstructability pattern. By utilizing the lattice of general structures, this can clearly be accomplished by a procedure which for any given general structure generates all its immediate refinements (or, alternatively, all immediate aggregates). In order to solve the reconstructability problem, such procedure must be combined with procedures through which structure hypotheses can be evaluated and compared with respect to the objectives of the reconstructability problem, and with

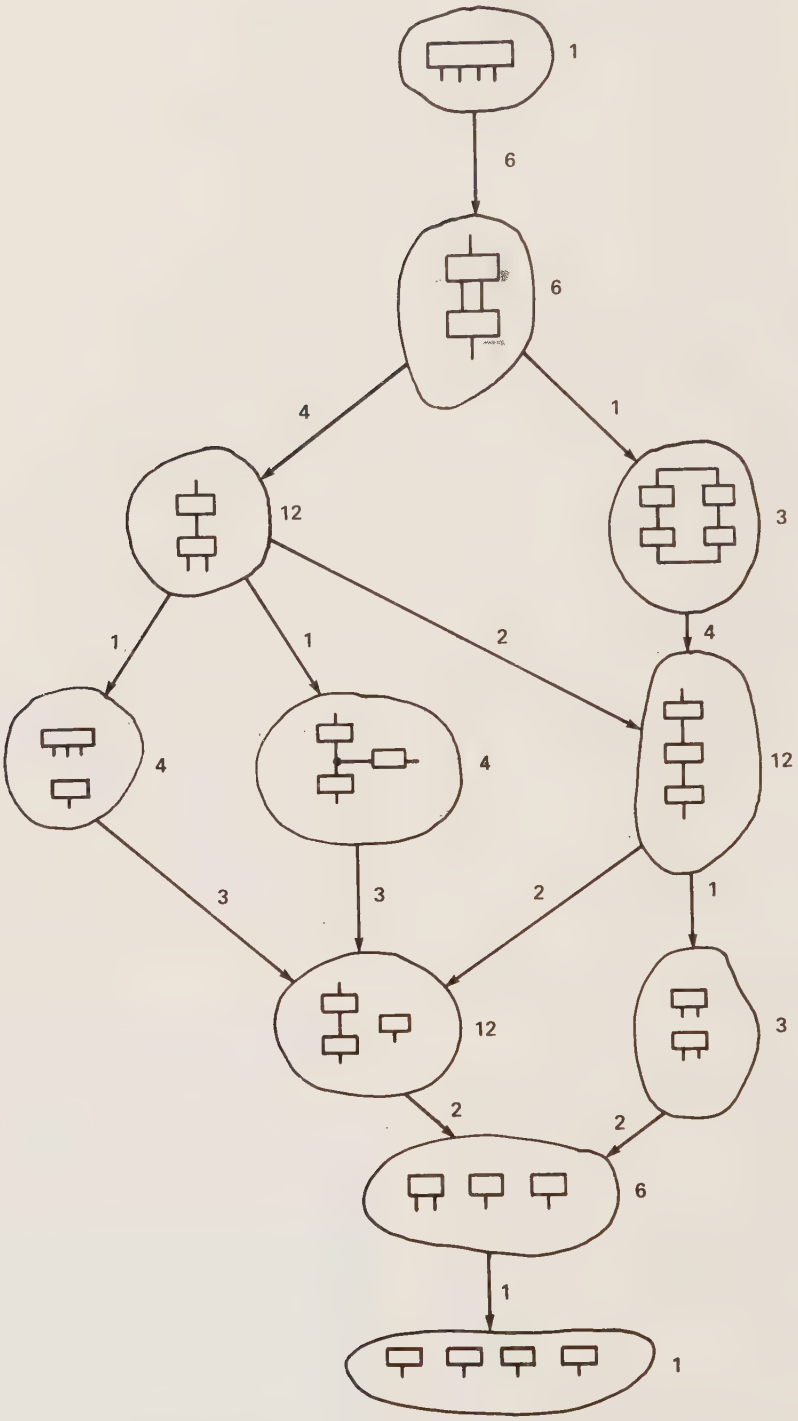

FIGURE 7 Lattice defined on S_c/i for 4 variables.

TABLE IV

n	1	2	3	4	5	6
$*\lvert S\rvert_n$	4	16	256	65,536	$\sim 4.3\times 10^9$	$\sim 1.8\times 10^{19}$
$*\lvert S_H\rvert_n$	1	4	104	32,256	$\sim 2.1\times 10^9$	$\sim 9\times 10^{18}$
$\lvert S_G\rvert_n$	1	2	9	114	7,199	?
$*\lvert S_C\rvert_n$	1	2	8	64	1,024	32,768
$*\lvert S_P\rvert_n$	1	2	8	64	1,024	32,768
$\lvert S_\beta\rvert_n$	1	2	8	61	762	14,554
$\lvert S_\gamma\rvert_n$	1	2	8	61	762	14,434

TABLE V

n	1	2	3	4	5	6
$*\lvert S/i\rvert_n$	4	12	80	3,984	$\sim 3.7\times 10^7$	$\sim 2.56\times 10^{16}$
$*\lvert S_H/i\rvert_n$	1	4	34	1,952	1.8×10^7	$\sim 1.3\times 10^{16}$
$\lvert S_G/i\rvert_n$	1	2	5	20	187	?
$*\lvert S_C/i\rvert_n$	1	2	4	11	34	156
$*\lvert S_P/i\rvert_n$	1	2	4	11	34	156
$\lvert S_\beta/i\rvert_n$	1	2	4	10	26	84
$\lvert S_\gamma/i\rvert_n$	1	2	4	10	26	83

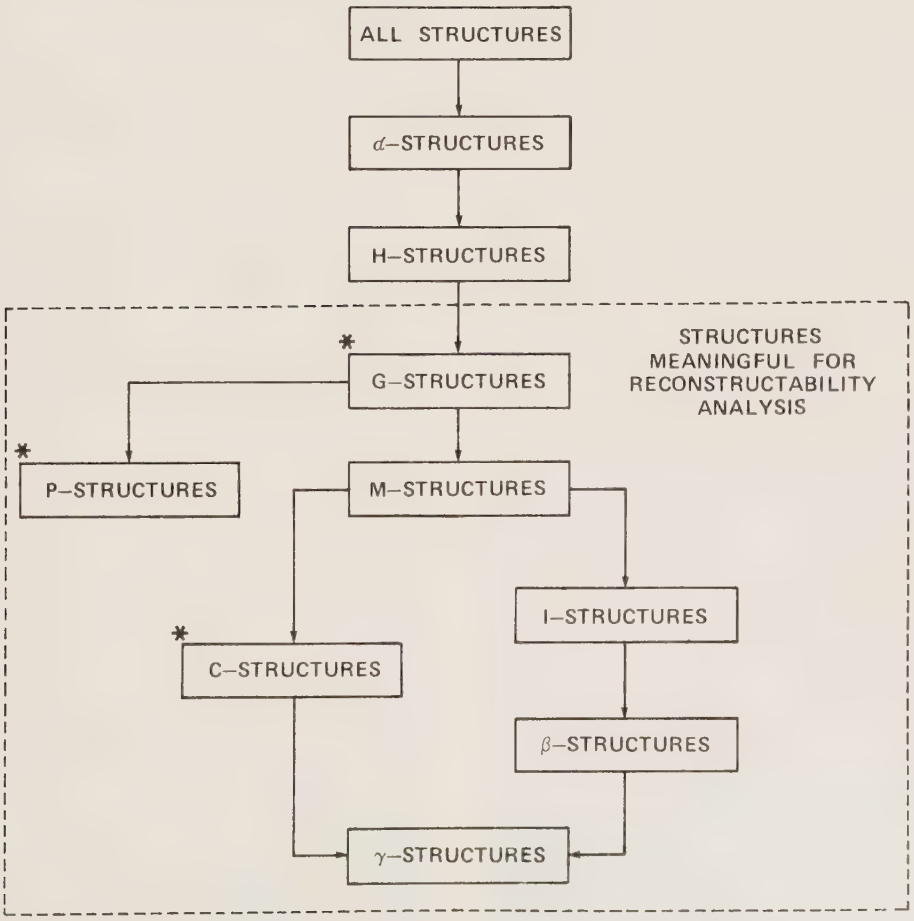

FIGURE 8 Hasse's diagram of the partial ordering among the considered classes of structures.

procedures through which it can be decided, at every stage of the reconstructability analysis, which of the generated and evaluated structures should be further refined.

This paper deals only with the procedure for generating structure hypotheses; it is referred to as the *generative procedure*. The evaluation and decision procedures will be subjects of other papers. To get some feeling about the nature of these latter procedures, the reader is referred to previous work, where the reconstructability analysis was restricted to β-structures.[10-12,24-28,36]

With the computational efficiency as well as application flexibility in mind, we propose that the overall generative procedure consist of a set of partial procedures, which are grouped into two subsets referred to as *basic procedures* and *supplementary procedures*.

The basic procedures are restricted either to the C-structures or to the P-structures; we refer to these two alternatives as *C-procedures* and *P-procedures*, respectively. Given a C-structure (or a P-structure), a C-procedure (or P-procedure) generates either all its immediate refinements or all its immediate aggregates. We call these two options the *refining* and *aggregating modes* of the basic procedures. Hence, there are four basic procedures: refining and aggregating C-procedures, and refining and aggregating P-procedures. Let these procedures be referred to as *RC-procedure*, *AC-procedure*, *RP-procedure* and *AP-procedure*, respectively; letters R and A are used here to indicate, respectively, the refining and aggregating modes.

Each C-structure, say C_i, generated by a C-procedure is viewed as a canonical representative of the r-equivalence class of general structures identified by the graph $\rho_i = r_C(C_i)$. This is justified by Theorem 1. Similarly, if a P-procedure is used, each generated P-structure is viewed as a canonical representative of the r-equivalence class of general structures identified by the graph $\rho_i = r_P(P_i)$. This is trivially justified by the fact that each P-structure is uniquely defined in terms of a graph.

The objective of the *supplementary procedures* is to generate all general structures in any given r-equivalence class characterized by a particular graph and either the corresponding canonical C-structure or the canonical P-structure. Similarly as with the basic procedures, the supplementary procedures involve two options referred to as refining and aggregating modes. Let these two procedures be referred to as *RS-procedure* and *AS-procedure*.

Given a general structure, say G_k, in an r-equivalence class represented by the canonical C-structure, say C_i, the *RS-procedure* generates all immediate refinements of G_k in the equivalence class. Since C_i is clearly the least refined structure in the equivalence class, the *RS*-procedure allows all general structures in the equivalence class to be generated, provided that it is initially applied to structure C_i and then repeatedly applied to structures generated in the previous step. If the equivalence class is represented by the canonical P-structure, say P_i, the *AS-procedure* is needed. It generates all immediate aggregates of G_k in the equivalence class. Since P_i is the most refined structure in the equivalence class, the *AS*-procedure allows all general structures in the given equivalence class to be generated, provided that it is initially applied to structure P_i and then repeatedly applied to structures generated in the previous step.

These procedures can be used *piece-meal* in the sense that the user may start with an initial C-structure C_i as a feasible hypothesis, analyze it and, if desirable, use the *RS*-procedure to generate and analyze all general structures in the r-equivalence class represented by C_i. Using both the *RC*-procedure and *AC*-procedure, the user can also explore the structure neighborhood of C_i and, if desirable, use the *RS*-procedure to investigate each of the r-equivalence classes represented by the C-structures in the neighborhood of C_i. In a similar way, P-structures and the refining and aggregating P-procedures can be utilized, if desirable; the *AS*-procedure is the relevant procedure in this case. Furthermore, an arbitrary general structure, which does not necessarily coincide with either of the canonical structures in its r-equivalence class, may be the desired starting point for an analysis. In such cases, the application of both the refining and aggregating supplementary procedures would allow the user to generate all general structures in the respective equivalence class. Then, the user can proceed with either mode of the C-procedures or, alternatively, the P-procedures.

The procedures can also be used in an *automatic fashion*. In that case, a number of meaningful alternatives of sequencing the procedures should be made available to the user to allow him to properly handle the various alternatives of the reconstructability problem.

If the user is interested solely in γ-structures, it is sufficient to consider only C-structures due to the fact that $S_\gamma \subseteq S_C$ (*Theorem* 2). Hence, only the C-procedures are needed in this case, augmented by some test through which it can be determined whether each generated C-structure is also a γ-structure.

If the user is interested in β, I or M-structures, both basic and supplementary procedures are needed, together with appropriate tests for properties of structures in these classes. Alternatively, specialized procedures for generating structures in these classes can be used, such as the one for β-structures which has been previously developed.[24-28]

The crux of the basic procedures is that at every stage we use both the C-structure (or P-structure) to be refined or aggregated and its graph representation, say C_i and $\rho_i = r_C(C_i)$ (or P_i and $\rho_i = r_P(P_i)$, respectively). We generate all immediate refinements or all immediate aggregates by operating on ρ_i and, then, make corresponding adjustments in C_i (or in P_i). The algorithmic schemes of the four basic procedures and two supplementary procedures are described as follows.

RC-procedure (refining C-procedure). *Given* C_k *and* $\rho_k = r_C(C_k)$, *to generate all immediate refinements of* C_k *in class* S_C: (1) exclude one edge (arc) from graph ρ_k, say edge (v_i, v_j), where $v_i \neq v_j$ and $(v_i, v_j) \in \rho_k$; (2) split each $E_a \in X$, where $X = \{E_a | E_a \in C_k, E_a \supseteq \{v_i, v_j\}\}$, into two subsets $E'_a = E_a - \{v_i\}$ and $E''_a = E_a - \{v_j\}$, and let Y denote the family of sets E'_a and E''_a for all $E_a \in X$: (3) determine $Y' = Y - \{E_\alpha | E_\alpha \in Y, \exists (E_b \in C_k - X)(E_\alpha = E_b)\}$ and record $(C_k - X) \cup Y'$ as an immediate refinement of C_k; (4) repeat steps (1)–(3) for all edges in ρ_k.

Remarks regarding the RC-procedure. The exclusion of one arc from ρ_k in step (1) produces the smallest possible changes in the cliques of ρ_k and, consequently, it is guaranteed that the C-structure representing the new graph is an immediate refinement of C_k. In steps (2) and (3), cliques and complete subgraphs are fully represented by the corresponding compatible subsets of variables. Only those cliques in ρ_k which contain the excluded arc are affected by its exclusion; each of them naturally breaks down into two smaller complete subgraphs described in step (2) of the procedure. Those complete subgraphs obtained in step (2) which are not cliques of the new graph $\rho_k - \{(v_i, v_j)\}$ are excluded in step (3). The number of immediate refinements of C_k is obviously equal to the number of arcs which can be excluded from the graph ρ_k.

Example of the RC-procedure. Let C_k and ρ_k specified in Figure 9 be given. First iteration: (1) edge (1,4) is excluded from ρ_k; (2) set $\{1,4\}$ is split into $\{1\}$ and $\{4\}$; (3) set $\{4\}$ is excluded since $\{4\} \subset \{2,3,4\}$ and the C-structure $C^r_{k.1}$ shown in Figure 9 is recorded as the first immediate refinement of C_k. Second iteration: (1) edge (2,3) is excluded; (2) set $\{2,3,4\}$ is split into sets $\{2,4\}$ and $\{3,4\}$; (3) no set is excluded and the C-structure $C^r_{k.2}$ (Figure 9) is recorded as the second immediate refinement of C_k. Third iteration: (1) edge (2,4) is excluded; (2) set $\{2,3,4\}$ is split into sets $\{2,3\}$ and $\{3,4\}$; (3) no set is excluded and $C^r_{k.3}$ (Figure 9) is recorded. Fourth iteration: (1) edge (3,4) is excluded; (2) set $\{2,3,4\}$ is split into sets $\{2,3\}$ and $\{2,4\}$; (3) no set is excluded and $C^r_{k.4}$ (Figure 9) is recorded. Since steps (1)–(3) were repeated for all edges $(v_i, v_j) \in \rho_k$ such that $v_i \neq v_j$, the procedure ends.

AC-procedure (aggregating C-procedure). *Given* C_k *and* $\rho_k = r_C(C_k)$, *to generate all immediate aggregates of* C_k *in class* S_C: (1) include one edge into graph ρ_k, say edge (v_i, v_j), where $v_i \neq v_j$ and $(v_i, v_j) \notin \rho_k$; (2) for each $E_x \in C_k$ such that $v_i \in E_x$ and for each $E_y \in C_k$ such that $v_j \in E_y$ construct the set $\{v_i, v_j\} \cup (E_x \cap E_y)$, let Z denote the family of all these sets, and let $C'_k = C_k \cup Z$; (3) for each $E_a \in C_k$ and each $E_b \in Z$, if $E_a \subset E_b$, then exclude E_a from C'_k, and for each distinct ordered pair E_α, $E_\beta \in Z$, if $E_\alpha \subset E_\beta$, then exclude E_α from C'_k; record the result of this step as an immediate aggregate of C_k; (4) repeat steps (1)–(3) for all edges which are not included in ρ_k.

Remarks regarding the AC-procedure. As in the RC-procedure, cliques and complete subgraphs are fully represented by the corresponding compatible subsets of variables. The inclusion of one missing edge, (v_i, v_j), into ρ_k produces the smallest possible changes in the cliques of ρ_k and, consequently, it is guaranteed that the C-structure representing the new graph is an immediate aggregate of C_k. There are no cliques in ρ_k which contain both v_i and v_j. Two cliques can potentially form a new clique, due to the inclusion of the arc (v_i, v_j), only when one of them contains v_i

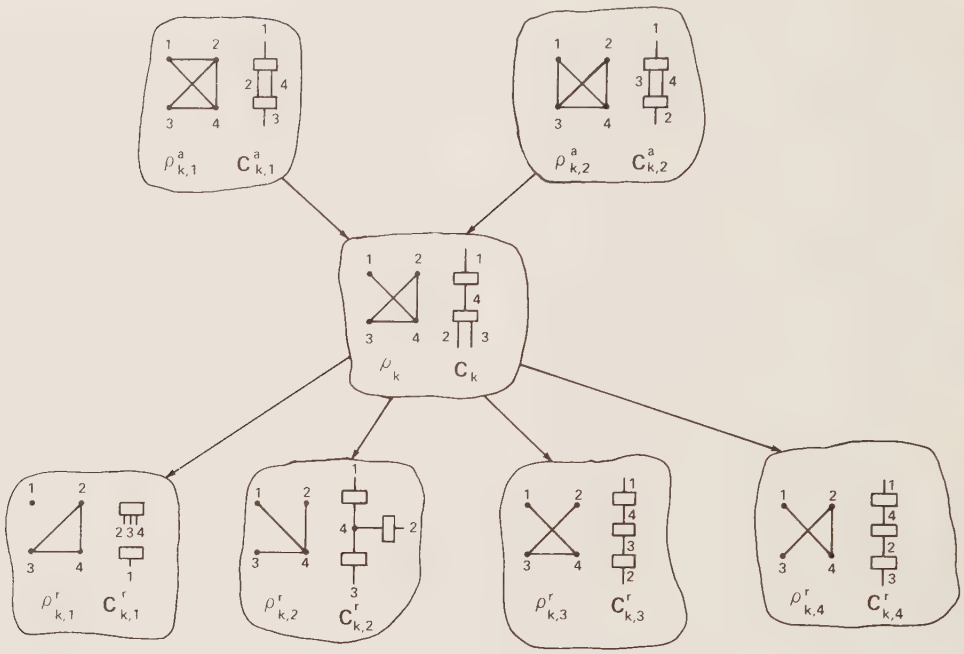

FIGURE 9 Structure neighborhood of the C-structure C_k; illustration of RC-procedure and AC-procedure (variables are represented by integers 1, 2, 3, 4).

and the other contains v_j. The potentially new clique is clearly formed by the set union of $\{v_i, v_j\}$ and the set of variables which are in both of the cliques. When a potentially new clique is formed, some of the cliques of ρ_k may be subsets of it and must be excluded because they are not cliques of the new graph $\rho_k \cup \{(v_i, v_j)\}$. Moreover, some of the potentially new cliques may be subsets of other potentially new cliques and must be excluded for the same reason. The number of immediate aggregates of C_k is clearly equal to the number of arcs which can be added to ρ_k.

Example of the AC-procedure. Let C_k and ρ_k specified in Figure 9 be given. First iteration: (1) edge $(1,2)$ is added to ρ_k; (2) the set $\{1,2\} \cup (\{1,4\} \cap \{2,3,4\}) = \{1,2,4\}$ is constructed; (3) set $\{1,4\} \in C_k$, which is a subset of set $\{1,2,4\}$, is excluded and $C^a_{k,1} = \{\{1,2,4\},\{2,3,4\}\}$ is recorded as the first aggregate of C_k. Second iteration: (1) edge $(1,3)$ is added to ρ_k; (2) the set $\{1,3\} \cup (\{1,4\} \cap \{2,3,4\}) = \{1,3,4\}$ is formed; (3) set $\{1,4\}$ is excluded and $C^a_{k,2}$ (Figure 9) is recorded as the second aggregate of C_k. Since steps (1)–(3) were repeated for all edges which are not included in ρ_k, the procedure ends.

RP-procedure (refining P-procedure). *Given* P *and* $\rho_k = r_P(P_k)$, *to generate all immediate refine-ments of* P_k *in class* S_P: (1) exclude one edge fro graph ρ_k, say edge (v_i, v_j), where $v_i \neq v_j$; (2) co struct $P'_k = P_k - \{\{v_i, v_j\}\}$; (3) let $X = \cup E_a$ for a $E_a \in P'_k$; if $v_i \notin X$ or $v_j \notin X$, include $\{v_i\}$ or $\{v_j\}$ P'_k, respectively, and record the result of this ste as an immediate refinement of P_k; (4) repeat step (1)–(3) for all edges in ρ_k.

AP-procedure (aggregating P-procedure). *Give* P_k *and* $\rho_k = r_P(P_k)$, *to generate all immediate ag-gregates of* P_k *in class* S_P: (1) include one edg into graph ρ_k, say edge (v_i, v_j), where $v_i \neq v_j$ an $(v_i, v_j) \notin \rho_k$; (2) construct $P'_k = P_k \cup \{\{v_i, v_j\}\}$; (3) $\{v_i\} \in P'_k$ or $\{v_j\} \in P'_k$, exclude $\{v_i\}$ or $\{v_j\}$ from P'_k respectively, and record the result of this step a an immediate aggregate of P_k; (4) repeat step (1)–(3) for all edges which are not included in ρ_k

The RP-procedure and AP-procedure a rather obvious and are illustrated by Figure 10.

RS-procedure (refining supplementary pr cedure). *Given* $G_{k,i}$ (*initially* $G_{k,0} = C_k$, *in case batch processing*) *and* $\rho_k = r_G(G_{k,i})$, *to generate a*

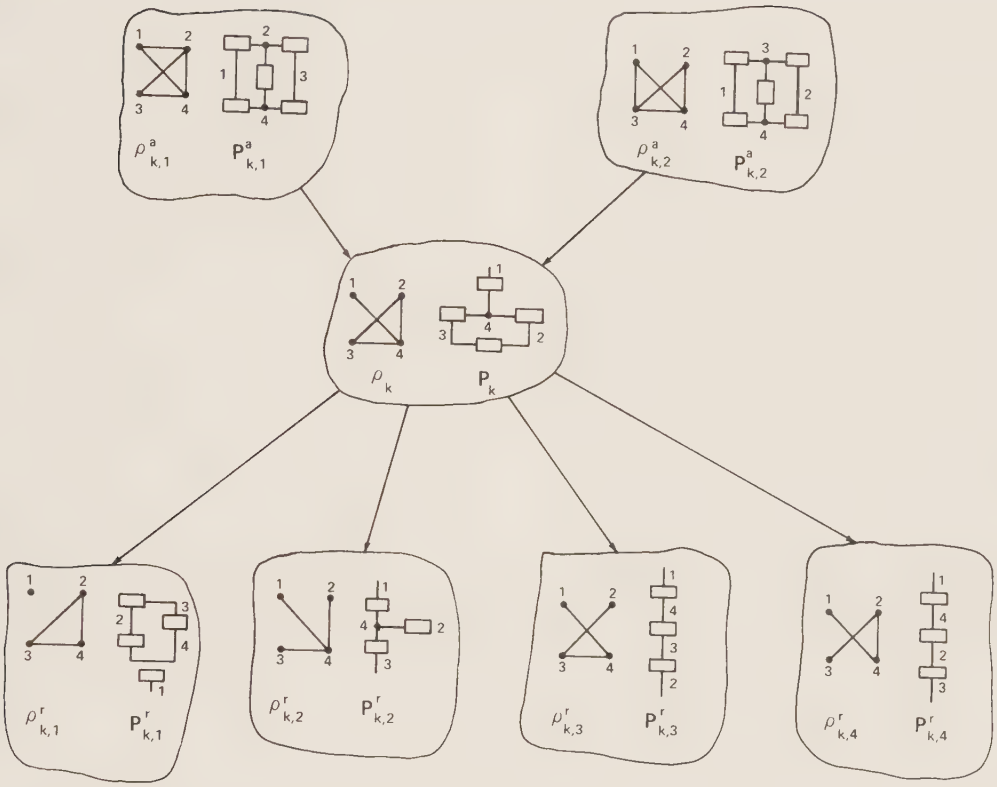

FIGURE 10 Structure neighbourhood of P-structure P_k; illustration of RP-procedure and AP-procedure (variables are represented by integers 1, 2, 3, 4).

immediate refinements of $G_{k,i}$ in class S_G and within the equivalence class characterized by graph ρ_k: (1) take one particular set $E_a \in G_{k,i}$ such that $|E_a| \geqq 3$ and replace $G_{k,i}$ with $G'_{k,i} = (G_{k,i} - \{E_a\}) \cup E'_a$, where $E'_a = \{E_{a,j} | E_{a,j} \subset E_a, |E_{a,j}| = |E_a| - 1\}$; (2) for each $E_{a,j} \in E'_a$ and for each $E_b \in G_{k,i} - \{E_a\}$, if $E_{a,j} \subset E_b$, then exclude $E_{a,j}$ from $G_{k,i}$, and record the result of this step as an immediate refinement of $G_{k,i}$; (3) repeat steps (1)–(2) for all sets $E_a \in G_{k,i}$ such that $|E_a| \geqq 3$.

Remarks regarding the RS-procedure. To generate an immediate refinement of general structure $G_{k,i}$, it is necessary to make sure that the modification of $G_{k,i}$ is minimal with respect to the partial ordering of structure refinements, which, in turn, is defined in terms of a subset relationship. This is guaranteed (i) by making a change only in one subset of variables in $G_{k,i}$, say $E_a \in G_{k,i}$, and (ii) by replacing E_a with all the largest proper subsets of E_a, i.e., with all subsets of E_a with cardinality $|E_a| - 1$. If any of these subsets is itself a subset of some other subsets of variables in $G_{k,i}$, then it must be excluded in order to preserve the properties of a general structure (step (2) in the RS-procedure). If $|E_a| \geqq 3$, then the replacement of E_a with all its largest proper subsets does not affect the graph $r_G(G_{k,i})$. On the other hand, if $|E_a| = 2$, say $E_a = \{v_i, v_j\}$, then the mentioned replacement changes the graph. This follows directly from the fact that if $E_a \in G_{k,i}$ and $|E_a| = 2$, then there is no $E_b \in G_{k,i}$ such that $E_a \subset E_b$. Hence, if E_a is replaced with $\{v_i\}$ and $\{v_j\}$, then the pair (v_i, v_j) is excluded from the graph and, consequently, the generated G-structure is not in the same r-equivalence class as $G_{k,i}$. We may conclude from these observations that we can obtain a procedure for generating all immediate refinements of $G_{k,i}$, regardless of the r-equivalence classes, simply by replacing the requirement that $|E_a| \geqq 3$ in step (1) of the RS-procedure by a weaker requirement

that $|E_a| \geqq 2$. Let such procedure be called the *refining general procedure* or **RG-procedure**.

Example of RS-procedure. Let $G_{k,i}$ and ρ_k specified in Figure 11 be given. First iteration: (1) set $\{1,2,3\}$ is replaced with sets $\{1,2\}$, $\{1,3\}$ and $\{2,3\}$; (2) set $\{2,3\}$ is excluded because $\{2,3\} \subset \{2,3,4\} \in G_{k,i}$ and $(G_{k,i})^r_1$, shown in Figure 11, is recorded as the first immediate refinement of $G_{k,i}$. Second iteration: (1) set $\{2,3,4\}$ is replaced with sets $\{2,3\}$, $\{2,4\}$ and $\{3,4\}$; (2) set $\{2,3\}$ is excluded and $(G_{k,i})^r_2$, shown in Figure 11, is recorded as the second immediate refinement. Since steps (1) and (2) were repeated for all $E_a \in G_{k,i}$ such that $|E_a| \geqq 3$, the procedure ends. If we allowed $|E_a| \geqq 2$ in step (1), the procedure would be modified into the *RG*-procedure and one more iteration of steps (1) and (2) is applicable in this example: (1) set $\{1,4\}$ is replaced with sets $\{1\}$ and $\{4\}$; (2) both sets $\{1\}$ and $\{4\}$ are excluded and $(G_{k,i})^r_3$ shown in Figure 11 is recorded as the third immediate refinement of $G_{k,i}$; observe that this immediate refinement is in a different r-equivalence class (represented by graph ρ'_k) than the given structure $G_{k,i}$.

***AS*-procedure** (aggregating supplementary procedure). *Given* $G_{k,i}$ (*initially* $G_{k,i} = P_k$, *in case of batch processing and* $\rho_k = r_G(G_{k,i})$, *to generate all immediate aggregates of* $G_{k,i}$ *in class* S_G *and within the r-equivalence class characterized by graph* ρ_k: (1) for each pair $E_a \in G_{k,i}$ and $v_\alpha \in V - E_a$, if $(v_\alpha, v_\beta) \in \rho_k$ for all $v_\beta \in E_a$, then record the pair E_a, $E'_a = E_a \cup \{v_\alpha\}$, and let X denote the set of all recorded pairs (E_a, E'_a); (2) for every two pairs $(E_a, E'_a) \in X$ and $(E_b, E'_b) \in X$, if $E'_a \supseteq E'_b$, then exclude (E_a, E'_a) from X; (3) for each pair (E_a, E'_a) left in X after executing step (2): if each subset of E'_a of cardinality $|E'_a| - 1$ is a subset of some $E_x \in G_{k,i} (E_x \neq E_a)$, then construct $G'_{k,i} = (G_{k,i} - Y) \cup \{E'_a\}$, where $Y = \{E_y | E_y \in G_{k,i}, E_y \subset E'_a\}$, and record $G'_{k,i}$ as an immediate aggregate of $G_{k,i}$; else, construct immediate aggregates—one of each of the subsets of E'_a of cardinality $|E'_a| - 1$

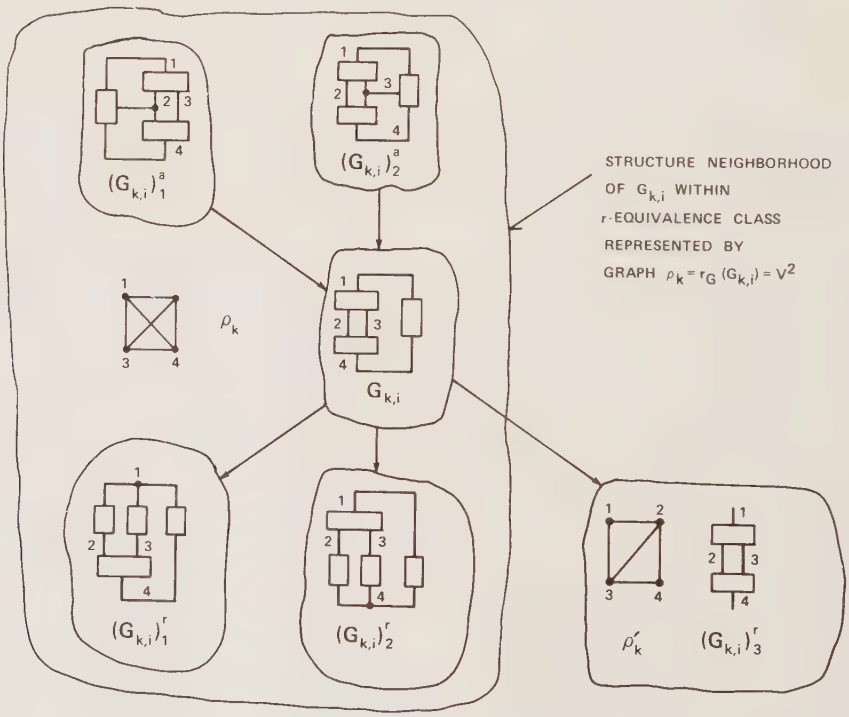

FIGURE 11 Complete structure neighborhood of general structure G_k, and structure neighborhood limited to r-equivalence class represented by graph $\rho_k = r_G(G_{k,i}) = V^2$; illustration of *RS*-procedure, *AS*-procedure and *RG*-procedure (variables are represented by integers 1, 2, 3, 4).

which is not a subset of some $E_x \in G_{k,i} (E_x \neq E_a)$—by adding that subset to $G_{k,i}$.

Remarks regarding AS-procedure. A minimal aggregating change associated with a particular $E_a \in G_{k,i}$ is made by replacing E_a with $E'_a = E_a \cup \{v_\alpha\}$ provided that the set represents a complete subgraph of ρ_k. If some of the replacements E'_a are supersets of other replacements, they are obviously not minimal aggregations and must be disregarded. Moreover, a replacement E'_a represents a minimal change provided that all subsets of E'_a of cardinality $|E'_a| - 1$ are included in the sets representing $G_{k,i}$. If some of them are not included in the sets of $G_{k,i}$, then the minimal aggregation is achieved by adding these sets to $G_{k,i}$, one at a time; when one of these sets is added to $G_{k,i}$, an immediate aggregate of $G_{k,i}$ is created. Observe that the *AS*-procedure can be extended into *aggregating general procedure* (or **AG-procedure**), through which all immediate aggregates of $G_{k,i}$ (regardless of the *r*-equivalence classes) are generated, by adding the following two steps into the *AS*-procedure: (4) include one edge into the graph ρ_k, say edge (v_i, v_j), such that $v_i \neq v_j$ and $(v_i, v_j) \notin \rho_k$, and construct $G'_{k,i} = G_{k,i} \cup \{\{v_i, v_j\}\}$; if sets $\{v_i\}$ or $\{v_j\}$ are included in $G'_{k,i}$, then exclude them and record the result of this step as an immediate aggregate of $G_{k,i}$; (5) repeat step (4) for all edges $(v_i, v_j) \notin \rho_k$.

Example of AS-procedure. Let $G_{k,i}$ specified in Figure 11 be given, which implies that $\rho_k = V^2$. (1) Four pairs can be formed for $G_{k,i}$: (i) $\{1,2,3\}$, 4; (ii) $\{2,3,4\}$, 1; (iii) $\{1,4\}$, 2; (iv) $\{1,4\}$, 3. These pairs form, respectively, the following sets E'_a: (i) $\{1,2,3,4\}$; (ii) $\{1,2,3,4\}$; (iii) $\{1,2,4\}$; (iv) $\{1,3,4\}$. (2) Set $\{1,2,3,4\}$ is excluded; sets $\{1,2,4\}$ and $\{1,3,4\}$ remain listed as replacements of set $\{1,4\}$ in $G_{k,i}$; as all subsets of each of these sets are included in $G_{k,i}$, two immediate aggregates of $G_{k,i}$ are generated: $(G_{k,i})^a_1$ and $(G_{k,i})^a_2$, specified in Figure 11.

When any of the described procedures is applied to several structures at the same level of refinement, the generated sets of immediate refinements or aggregates may overlap and, consequently, a procedure has to be included through which duplicates are excluded. This procedure as well as implementation details of the eight procedures presented in this paper, their more specific justification, estimates of their computational complexity, and their integration in two overall software packages, one for interactive processing and one for batch processing, will be covered in another paper.

7 SUMMARY AND CONCLUSIONS

Our approach to the reconstructability problem, as discussed in Sections 1 and 2, requires that the following three diverse kinds of procedures become embedded in the overall reconstructability analysis: (i) procedures for generating, in an orderly fashion, all desirable structure hypotheses; (ii) procedures for evaluating and comparing the generated structure hypotheses; (iii) procedures through which it can be decided, at every stage of the reconstructability analysis, which of the generated and evaluated structure hypotheses should be used as a basis for further analysis. This paper is devoted solely to the problem of generating structure hypotheses. The other two problems, associated with the evaluation and comparison procedures and the decision making procedures, will be covered in separate papers.

The problem of generating structure hypotheses is satisfactorily solved, at the highest desirable level of generality, in terms of the six generative procedures, each of which takes advantage of the fact that the respective set of structure hypotheses forms a lattice. The four basic procedures can be viewed as *macroprocedures* in the sense that they operate on *r*-equivalence classes of general structures rather than on individual structures. The two supplementary procedures are then viewed as *microprocedures* in the sense that they allow the generation of individual structures in each desirable *r*-equivalence class. This combination of macroprocedures and microprocedures makes the generation of structure hypotheses computationally highly efficient. Whenever desirable, the user has also the option to use the *RG*-procedure and/or *AG*-procedure. These procedures allow him to explore the complete structure neighborhood of a given general structure independent of consideration of the *r*-equivalence classes.

It seems that the reconstructability problem was first recognized by Ashby in the early sixties.[3,4] Although he indicates in his papers various possible extensions, his formulation of the problem is restricted to relations which have no

measure associated with them, and his *constraint analysis* is restricted to the reconstruction of relations of order n from all projections of dimensions $n-1$, $n-2, \ldots, 2$, respectively.

The approach to the reconstructability problem which is adopted in this paper was first suggested by Klir in 1976[24] and developed later by Klir and Uyttenhove for the β-structures and relations associated with probability distributions.[26-28, 36] A few applications of the reconstructability analysis, limited to the β-structures, have also been explored.[13, 18, 25, 27, 36]

Stimulated primarily by the work of Ashby,[3-6] Conant[19, 20] and Watanabe,[37] as well as the formulation of the reconstructability problem proposed by Klir,[24] Broekstra and Krippendorff have also been working on this problem recently and are developing the application of some concepts from information theory.[10-12, 29] This work is compatible with the theoretical basis presented here and we intend to integrate this and other information theoretic work into our further development of reconstructability analysis, as well as into the overall systems problem solving framework (GSPS) referred to in Section 1.[14]

The reconstructability problem is undoubtedly one of the most fundamental problems in general systems research. It has recently been emerging, in one form or another, in so many diverse application areas that their survey would be better reserved for a separate paper. As just one example of such application areas, with growing significance in computer science, we merely mention studies of relational data structures,[16, 17, 21, 34] particularly those which allow the general framework developed by Bang and Yeh.[7, 8, 38]

APPENDIX A

Tables VI, VII, VIII contain complete descriptions of the lattices of i-equivalence classes of general structures for $n = 3, 4, 5$, respectively.

Each i-equivalence class of general structures is represented in the tables by one particular general structure, a *representative* of the respective i-equivalence class. The representatives are specified in the fourth column of each table in terms of families of subsets of the respective variables. variables are denoted by integers and their subsets in each family are separated by slashes. The remaining columns in the tables have the following meaning: g is an identifier of i-equivalence classes of general structures; k is an identifier of r-equivalence classes; j is an identifier by which i-equivalence classes are distinguished in each r-equivalence class; $\#g$ denotes the number of distinct general structures in each i-equivalence class; $s(g)$ stands for the set of i-equivalence classes which are immediate successors (refinements) of the i-equivalence class identified by g. $\#s(g)$ denotes the set of numbers of immediate successors of i-equivalence class g, one for each of the i-equivalence classes in $s(g)$; individual i-equivalence classes in each set $s(g)$ are listed (in terms of their identifiers) together with the corresponding numbers in set $\#s(g)$ as pairs separated by slashes. Colons in Table VIII separate immediate successors in the same r-equivalence class from those in different classes.

Let $\#s(g, g')$ denote the number of immediate successors (refinements) of each general structure of the i-equivalence class g in the i-equivalence class g' and let $\#p(g', g)$ denote the numbers of immediate predecessors (aggregates) of each general structure of the i-equivalence class g' in the i-equivalence class g. Then, by simple considerations, we find that

$$\#g \cdot \#s(g, g') = \#g' \cdot \#p(g', g)$$

Hence,

$$\#p(g', g) = \frac{\#g \cdot \#s(g, g')}{\#g'}$$

TABLE VI
Description of the lattice $(S_G/i, \leqq)$ for 3 variables.

g	k	j	Subsystems	#g	s(g)/#s(g)
0	0	0	123	1	1/1
1		1	12/13/23	1	2/3
2	1	0	12/23	3	3/2
3	2	0	12/3	3	4/1
4	3	0	1/2/3	1	None

TABLE VII
Description of the lattice $(S_G/i, \leqq)$ for 4 variables.

g	k	j	Subsystems	$\#g$	$s(g)/\#s(g)$		
0	0	0	1234	1	1/1		
1		1	123/124/134/234	1	2/4		
2		2	123/124/134	4	3/3		
3		3	123/124/34	6	4/2	6/1	
4		4	123/14/24/34	4	5/1	7/3	
5		5	12/13/14/23/24/34	1	8/6		
6	1	0	123/124	6	7/2		
7		1	123/14/24	12	8/1	9/2	
8		2	12/13/14/23/24	6	10/4	11/1	
9	2	0	123/14	12	10/1	12/1	
10		1	12/13/14/23	12	13/1	14/1	15/2
11	3	0	13/14/23/24	3	15/4		
12	4	0	123/4	4	13/1		
13		1	12/13/23/4	4	16/3		
14	5	0	12/13/14	4	16/3		
15	6	0	13/14/23	12	16/2	17/1	
16	7	0	13/14/2	12	18/2		
17	8	0	14/23	3	18/2		
18	9	0	14/2/3	6	19/1		
19	10	0	1/2/3/4	1	None		

TABLE VIII
Description of the lattice $(S_G/i, \leqq)$ for 5 variables.

g	k	j	Subsystems	$\#g$	$s(g)$	$\#s(g)$			
0	0	0	12345	1	1/1				
1		1	1234/1235/1245/1345/2345	1	2/4				
2		2	1235/1245/1345/2345	5	3/4				
3		3	234/1345/1245/1235	10	4/3	5/1			
4		4	123/235/1245/1345/234	10	6/3	7/2			
5		5	1235/1245/1345	10	6/3				
6		6	235/1245/1345/234	30	8/2	9/2			
7		7	124/125/245/123/235/1345/234	5	10/6	11/1			
8		8	1245/1345/234	30	12/2	13/1			
9		9	124/125/245/235/1345/234	30	14/4	15/1	16/1		
10		10	125/245/123/235/1345/234	30	16/1	17/2	18/2	19/1	
11		11	134/135/145/345/124/125/245/123/235/234	1	16/10				
12		12	124/125/245/1345/234	60	20/2	21/1	22/1	23/1	
13		13	23/1245/1345	10	23/2	: 61/1			
14		14	125/245/235 1345/234	60	20/2	21/1	22/1	23/1	
15		15	124/125/235/1345/234	15	20/4	24/1			
16		16	134/135 145/345/124/125 245/235/234	10	22/6	24/3			
17		17	245/123 235/1345/234	60	20/2	21/1	22/1	23/1	
18		18	125/123/235/1345/234	60	20/2	21/1	22/1	23/1	
19		19	125/245/123 1345/234	15	20/4	24/1			
20		20	125/245/1345/234	60	25/2	26/1	27/1		
21		21	124/245/1345/234	20	25/3	28/1			
22		22	134/135/145/345/124/125/245/234	30	26/4	28/2	29/1	30/1	
23		23	23/124/125/245/1345	20	25/3	29/1	: 62/1		
24		24	134/135/145/345/124/125/235/234	15	26/4	30/4			
25		25	23/1345/245/125	60	31/2	32/1	: 64 1		
26		26	134/135/145/345/234/245/125	60	32/2	33/1	34/2	35/2	
27		27	234/1345/125	15	31/2	33/1			
28		28	134/135/145/345/234/245/124	10	32/3	35/3	36/1		
29		29	23/245/125/124/345/145/135/134	10	37/6	38/1	: 63/1		
30		30	234/245/125/124/345/135/134	10	34/4	35/2	38/1		

TABLE VIII (Continuation)

g	k	j	Subsystems	$\#g$	$s(g)/\#s(g)$
31		31	12/245/1345/23	30	39/1 40/1 : 66/2
32		32	134/135/145/345/125/245/23	60	39/2 41/2 42/1 43/1 : 65/1
33		33	125/234/345/145/135/134	15	39/2 44/4
34		34	125/245/234/145/135/134	60	41/1 42/2 44/2 45/1
35		35	125/245/234/345/145/134	60	42/2 43/1 44/2 46/1
36		36	124/245/234/345/145/134	5	43/6
37		37	135/145/345/124/125/245/23	60	39/2 41/2 42/1 43/1 : 65/1
38		38	134/135/345/124/125/245/23	10	42/6 : 65/1
39		39	134/135/145/345/23/245/12	30	47/1 48/2 49/2 : 70/2
40		40	24/25/23/1345/12	5	47/1 : 71/4
41		41	23/245/125/145/135/134	60	48/2 49/2 50/1 : 68/1
42		42	23/245/125/345/135/134	60	48/2 50/2 51/1 : 69/1
43		43	23/245/125/345/145/135	30	49/4 51/1 : 67/1
44		44	135/145/345/234/125	30	48/1 49/1 50/2 52/1
45		45	134/135/234/245/125	12	50/5
46		46	35/134/145/234/245/125	60	48/2 50/2 51/1 : 69/1
47		47	24/25/12/23/345/145/135/134	5	53/4 : 77/4
48		48	12/245/23/145/135/134	60	53/1 54/2 55/1 : 74/1 76/1
49		49	12/245/23/345/145/134	60	53/1 54/2 56/1 : 72/2
50		50	134/135/125/245/23	60	54/2 55/2 : 75/1
51		51	14/135/345/125/245/23	15	54/4 : 73/2
52		52	125/234/145/135	10	55/3 56/1
53		53	135/145/345/23/12/25/24	20	57/3 : 81/3 83/1
54		54	34/135/145/23/245/12	60	57/2 58/1 : 79/2 80/1
55		55	134/135/23/245/12	30	57/1 58/2 : 82/2
56		56	13/145/345/23/245/12	10	57/3 : 78/3
57		57	24/25/12/23/145/135/34	30	59/2 : 84/2 85/1 86/2
58		58	14/12/245/23/135/34	15	59/2 : 87/4
59		59	13/35/34/145/23/12/25/24	10	60/1 : 88/6 89/1
60		60	14/15/45/24/25/12/23/34/35/13	1	:90/10
61	1	0	1245/1345	10	62/2
62		1	134/135/345/1245	20	63/1 64/3
63		2	124/125/145/245/345/135/134	10	65/7
64		3	1245/135/134	60	65/1 66/2
65		4	135/345/245/145/125/124	60	67/1 68/2 69/1 70/2
66		5	34/135/1245	60	70/1 71/1 : 91/1
67		6	125/145/245/345/135	30	72/4 73/1
68		7	124/145/245/345/135	30	72/2 74/2 75/1
69		8	124/125/245/345/135	60	73/1 75/2 76/2
70		9	34/124/125/145/245/135	60	72/2 74/1 76/1 77/1 : 92/1
71		10	13/35/1245/34	20	77/1 : 93/3
72		11	13/345/245/145/125	120	78/1 79/1 80/1 81/1 : 95/1
73		12	14/135/345/245/125	30	79/4 : 112/1
74		13	34/135/245/145/124	60	80/2 81/1 82/1 : 96/1
75		14	135/345/245/124	60	79/2 82/2
76		15	13/345/245/125/124	60	79/2 82/1 83/1 : 96/1
77		16	13/35/245/145/125/124/34	20	81/3 83/1 : 94/3
78		17	12/145/245/345/13	30	84/2 85/1 : 99/2
79		18	14/125/245/345/13	120	85/1 86/1 87/1 : 100/1 113/1
80		19	24/125/145/345/13	60	84/2 87/1 : 100/2
81		20	34/35/125/145/245/13	60	84/2 86/1 : 97/2 98/1
82		21	124/245/135/34	60	86/1 87/2 : 101/1
83		22	34/35/124/125/245/13	20	86/3 : 98/3
84		23	34/35/13/245/145/12	60	88/1 89/1 : 102/1 103/2 105/1
85		24	14/15/13/345/245/12	30	28/2 : 105/1 107/1 115/2
86		25	34/35/13/245/125/14	60	88/2 : 103/2 104/1 115/1
87		26	24/13/345/125/14	60	88/2 : 106/2 114/1
88		27	14/15/12/245/13/35/34	60	90/1 : 108/1 109/2 110/1 116/2
89		28	24/25/12/145/13/35/34	10	90/1 : 108/6
90		29	24/25/45/34/35/13/12/15/14	10	:111/6 117/3

TABLE VIII (Continuation)

g	k	j	Subsystems	$\#g$	$s(g)/\#s(g)$
91	2	0	1245/135	30	92/1 93/1
92		1	124/125/145/245/135	30	94/1 95/2 96/2
93		2	13/35/1245	30	94/1 : 128/2
94		3	13/35/245/145/125/124	30	97/2 98/2 : 129/2
95		4	135/145/125/124	60	97/1 99/1 100/2
96		5	135/245/125/124	60	98/1 100/2 101/1
97		6	125/145/245/35/13	60	102/1 103/2 : 130/1 131/1
98		7	124/145/245/35/13	60	103/2 104/1 : 131/2
99		8	24/125/145/135	30	102/1 105/2 : 124/1
100		9	25/124/145/135	120	103/1 106/1 107/1 : 118/1
101		10	124/245/135	30	104/1 106/2
102		11	24/13/35/145/125	30	108/2 : 108/2 125/1 132/2
103		12	14/13/35/245/125	120	108/1 109/1 : 119/1 132/1 133/1
104		13	15/13/35/245/124	30	109/2 : 133/2 137/1
105		14	14/45/135/125/24	60	108/1 110/1 : 119/2 125/1
106		15	45/135/124/25	60	109/1 110/1 : 120/2
107		16	12/24/135/145/25	60	108/1 110/1 : 119/2 125/1
108		17	12/25/145 35/13/24	60	111/1 : 122/2 126/1 134/2
109		18	12/15/245/35/13/14	60	111/1 : 121/1 134/1 135/1 138/1
110		19	12/25/24/135/45/14	30	111/1 : 121/4 126/1
111		20	14/15/45/24/13/35/25/12	30	:123/4 127/1 136/2 139/1
112	3	0	125/135/245/345	15	113/4
113		1	34/245/135/125	60	114/1 115/2 : 118/1
114		2	12/135/245/34	30	116/2 : 120/2
115		3	13/35/125/245/34	60	116/2 : 119/2 137/1
116		4	24/25/45/34/135/12	60	117/1 : 121/2 122/1 138/2
117		5	13/15/35/12/34/45/25/24	15	:123/4 139/4
118	4	0	125/135/245	60	119/2 120/1
119		1	24/45/135/125	120	121/1 122/1 : 141/1 148/1
120		2	12/245/135	60	121/2 : 145/1
121		3	12/25/135/45/24	120	123/1 : 143/1 146/1 149/1 157/1
122		4	13/35/125/45/24	60	123/1 : 142/2 149/2
123		5	13/15/35/24/45/25/12	60	:144/2 147/1 150/2 158/2
124	5	0	125/135/145	10	125/3
125		1	14/45/135/125	30	126/2 : 148/2
126		2	12/25/135/45/14	30	127/1 : 149/4
127		3	12/25/45/14/13/15/35	10	:140/1 150/6
128	6	0	1245/13	20	129/1 : 151/1
129		1	124/125/145/245/13	20	130/1 131/3 : 152/1
130		2	13/145/125/124	20	132/3 : 153/1
131		3	13/245/125/124	60	132/1 133/2 : 153/1
132		4	24/125/145/13	60	134/2 : 148/1 154/1
133		5	14/125/245/13	60	134/1 135/1 : 141/1 148/1
134		6	14/45/13/125/24	60	136/1 : 142/1 149/2 155/1
135		7	12/15/13/245/14	20	136/1 : 143/3 155/1
136		8	12/15/25/24/13/45/14	20	:144/2 150/4 156/1
137	7	0	12/135/24/345	30	138/2 : 141/2
138		1	34/45/24/135/12	60	139/1 : 142/1 143/1 157/2
139		2	34/45/24/12/13/15/35	30	:140/1 144/2 158/4
140	8	0	12/13/14/25/35/45	10	:159/6
141	9	0	125/13/245	60	142/1 143/1 : 162/1
142		1	24/45/13/125	60	144/1 : 160/2 163/1
143		2	12/15/245/13	60	144/1 : 163/1 165/2
144		3	24/45/13/12/15/25	60	:159/1 161/1 164/1 166/2
145	10	0	135/245	15	146/2
146		1	24/25/45/135	30	147/1 : 165/2 167/1
147		2	24/25/45/13/15/35	15	:166/4 168/2
148	11	0	125/245/35	60	149/2 : 162/1
149		1	24/45/35/125	120	150/1 : 160/1 163/1 167/1
150		2	24/45/35/12/15/25	60	:159/1 161/2 164/1 168/2

TABLE VIII (Continuation)

g	k	j	Subsystems	$\#g$	$s(g)/\#s(g)$
151	12	0	1245/3	5	152/1
152		1	124/125/145/245/3	5	153/4
153		2	145/125/124/3	20	154/3
154		3	24/125/145/3	30	155/2 : 162/1
155		4	14/45/125/24/3	20	156/1 : 163/3
156		5	14/45/24/12/15/25/3	5	:164/6
157	13	0	12/135/24/45	60	158/1 : 160/1 165/2
158		1	12/24/45/13/15/35	60	:159/2 161/1 166/2 169/1
159	14	0	12/13/15/24/45	60	:170/2 173/2 174/1
160	15	0	124/13/45	60	161/1 : 175/2
161		1	13/45/12/14/24	60	:170/1 173/2 176/2
162	16	0	125/245/3	30	163/2
163		1	45/24/125/3	60	164/1 : 175/2
164		2	45/24/12/15/25/3	30	:174/1 176/4
165	17	0	124/13/35	60	166/1 : 171/1 175/1
166		1	13/35/12/14/24	60	:170/2 172/1 173/1 176/1
167	18	0	124/13/15	30	168/1 : 175/2
168		1	13/15/12/14/24	30	:173/2 176/2 177/1
169	19	0	12/13/24/35/45	12	:170/5
170	20	0	12/13/24/45	60	:178/2 182/2
171	21	0	124/35	10	172/1 : 179/1
172		1	35/12/14/24	10	:180/1 182/3
173	22	0	12/23/24/45	60	:178/2 181/1 182/1
174	23	0	12/15/24/45/3	15	:178/4
175	24	0	12/245/3	60	176/1 : 179/1
176		1	12/24/25/45/3	60	:178/2 180/1 181/1
177	25	0	13/23/34/35	5	:181/4
178	26	0	12/24/45/3	60	:183/2 184/1
179	27	0	124/3/5	10	180/1
180		1	12/14/24/3/5	10	:183/3
181	28	0	12/24/25/3	20	:184/3
182	29	0	12/24/35	30	:183/1 184/2
183	30	0	12/13/4/5	30	:185/2
184	31	0	12/45/3	15	:185/2
185	32	0	12/3/4/5	10	:186/1
186	33	0	1/2/3/4/5	1	None

This formula allows us to calculate, using only information in Tables VI–VIII, the number of immediate predecessors in individual i-equivalence classes g for each given general structures of the i-equivalence class g'. For example, if $g=8$ and $g'=10$ in Table VII, then $\#g=6$, $\#g'=12$, $\#s(g,g')=4$, and

$$\#p(g',g)=\frac{6\times 4}{12}=2.$$

APPENDIX B

In order to prove *Theorem 2:* $S_\gamma \subseteq S_C$, we take advantage of the following *Lemma.*

LEMMA *Let* $M_i \in S_M$ *and let* $r_M(M_i) = r_M(M_i \cup \{X\})$, *where* X *is a maximal compatible of* $r_M(M_i)$ *such that* $X \notin M_i$. *Then there exist three variables in* X, *say* v_p, v_q, v_r, *and three subsystems in* M_i, *say* E_a, E_b, E_c, *such that* $\{v_p, v_q\} \subseteq E_a$, $\{v_p, v_r\} \subseteq E_b$, $\{v_q, v_r\} \subseteq E_c$, *and* $v_r \notin E_a$, $v_q \notin E_b$, $v_p \notin E_c$.

Proof It is obvious that X must be such that $|X| \geq 3$; indeed if $|X| < 3$ and $X \notin M_i$, then $r_M(M_i) \neq r_M(M_i \cup \{X\})$. Let $E_a \in M_i$ contain at least two variables which are included in X. Since it cannot contain all variables in X, there must be some subsystem of M_i, say $E_b \in M_i$, which contains variables x and y such that $x \in X - E_a$ and $y \in E_a \cap X$. We consider now the following cases: (1) Let $(X - E_a) \subseteq E_b$. Then there must be

some variable in $E_a \cap X$, say $z \in E_a \cap X$, such that $z \notin E_b$ (else $X \subseteq E_b$ and, hence, X is not a maximal compatible). Consequently, there must be a third subsystem, say $E_c \in M_i$, such that $z \in E_c$ and $x \in E_c$. Hence, $\{y, z\} \subseteq E_a$, $\{x, y\} \subseteq E_b$, $\{x, z\} \subseteq E_c$ and the lemma is proven. (2) Let $\{X - E_a\} \not\subseteq E_b$ and let $(E_a \cap X) \not\subseteq (E_b \cap X)$. Then $E_a \cap X$ contains a variable, say $z \in E_a \cap X$, such that $z \notin E_b$, and M_i contains a subsystem, say $E_c \in M_i$, such that $x \in E_c$ and $z \in E_c$; this proves the lemma. (3) Let $(X - E_a) \not\subseteq E_b$ and $(E_a \cap X) \subseteq (E_b \cap X) \subset X$. Then there must exist $v \in X$ and $E_c \in M_i$ such that $v \in E_c$ and $v \notin E_b$. Then the arguments given in (1) and (2) for E_a, E_b and x hold for E_b, E_c and v. Then, either the lemma is proven or we must consider another variable in X, say w, and another subsystem of M_i, say $E_d \in M_i$, so that arguments (1) and (2) hold for E_c, E_d and w. Since X is a finite set, this process must terminate and the lemma is proven.

THEOREM 2 $S_\gamma \subseteq S_C$

Proof Let M_i which satisfies the Lemma be a γ-structure. It follows from the Lemma that (i) $\{v_p, v_q\} \subseteq E_a$ and (ii) both $E_b \in (M_i - E_{p,q})$ and $E_c \in (M_i - E_{p,q})$, where $E_{p,q}$ has the same meaning as in the definition of S_γ. It follows from (ii) that the pair (v_p, v_q) is an element of the transitive closure of $r(M_i - E_{p,q})$. Hence, M_i is not a γ-structure. This directly implies the statement: if $M_i \notin S_C$, then $M_i \notin S_\gamma$, or an equivalent statement, if $M_i \in S_\gamma$, then $M_i \in S_C$.

We also observe that $S_\gamma \subset S_C$ for structures with at least four variables. Indeed, if $V = \{v_1, v_2, \ldots, v_n\}$ and $n \geq 4$, then there are C-structures which are not γ-structures, e.g., $\{\{v_1, v_2\}, \{v_2, v_3\}, \ldots, \{v_n, v_1\}\}$; if $n \leq 3$, then all C-structures are also γ-structures (see Figure 3).

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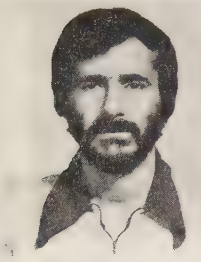

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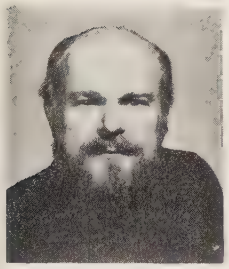

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OLD TRENDS AND NEW TRENDS IN GENERAL SYSTEMS RESEARCH†

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Ludwig von Bertalanffy, as well as many of the other pioneers of systems theory, was motivated in large part by the problems of theoretical biology. This fact reflects the unique role which biology plays in the sciences. The character of biological explanations, and the relation of organic problems to other realms of human activity, is reviewed in this context, and some of the fundamental remaining problems are pointed out. It is suggested that these problems will play a dominant role in systems research in the future, as they have in the past.

INDEX TERMS: Biological systems, mathematics, general systems research, systems model.

It is a great honor for me to have been invited to present this year's Ludwig von Bertalanffy Memorial Lecture. I was privileged to have known Ludwig von Bertalanffy personally in the last few years of his life, when he came to the State University of New York at Buffalo in the late 1960's. Although his appointment was in the Faculty of Social Sciences, his offices were located in the Center for Theoretical Biology, of which I was then Assistant Director. I cannot imagine a more fitting arrangement; von Bertalanffy's roots were solidly anchored in the theory of biological systems, of which he initiated some of the deepest pioneering studies; from these roots grew unique insights into the character of social and behavioral systems, as well as the creation of a General Systems Theory which bound all these activities together into an integrated intellectual edifice of unique amplitude and power.

I myself am primarily a theoretical biologist. My experience convinces me that the study of organic phenomena forces one down the path that von Bertalanffy was among the first to illuminate, and it is therefore no accident that the pioneers of systems theory were also deeply concerned with theoretical biology. For biology occupies a unique position, at the confluence of all sciences and technologies. In its microscopic realms it looms on the horizon of the physicist and chemist; at macroscopic levels organisms provide examples of control of complex behaviors which have inspired technologists, scientists and philosophers from Leonardo to Norbert Wiener and beyond; and organisms are both examples of and constituents of higher-order social, political and economic structures. As a result, there is no advance in any science which is without impact on biology; conversely, any development in biology tends to radiate out into every corner of natural philosophy. To understand the phenomena of organic nature *per se*, and to understand how they are related to other scientific realms, is one of the main forces which led to the creation of a theory of general systems, and determined its shape and thrust.

Ironically, it is only during the past few hundred years, since the rise of empiricism and reductionism, that we have come to recognize that life requires explanation. Before then, back to the dawn of thought, life was regarded as itself an explanatory principle, or as a primitive, irreducible feature of the world. The diagnostic of life was the capacity for autonomous movement; therefore any entity exhibiting autonomous movement was to that extent alive, and the spirit which animated it explained its movement. Consequently the stars and planets were alive; the sun, the moon and the clouds were alive. Moreover, in earliest times as well as in the present, man has striven to explain what he does not understand in terms of what he does (or

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thinks he does). Initially, man had no understanding of what we would now call inanimate nature; what he really understood was his society. He knew what it was for; he knew his place in it; he knew how it worked. Therefore, it was not surprising that he would attempt to explain natural phenomena in terms of a social or familial organization of vital units, not unlike the one he knew so well. Thus were born the various pantheons which were the General System Theories of their day, in which natural phenomena were considered to be understood when they were appropriately personified.

All of this was exorcised with astonishing rapidity at the beginning of the 18th century, although of course its basis had been eroding for a long time before that. The ultimate event in the banishment of life as an explanatory principle was probably the publication of Newton's *Principia*; the continuing influence of this book on the thought of the world is impossible to overestimate (although, like most mathematical books, it has not been widely read). For Newton was able to show that, on the basis of a small number of very simple universal laws, it was possible to explain, and to predict, all the celestial motions which had previously been regarded as autonomous or volitional. Newton also provided, in his development of the differential and integral calculus, the technical tools through which such predictions could be made, thereby inevitably tying natural phenomena to mathematical representation through appropriate symbolic encoding.

Newton's analysis was universally valid for any system consisting of material particles. Therefore, insofar as any natural systems can be regarded as consisting of material particles, the laws of Newtonian dynamics must be satisfied. The consequences of this are profound and immediate; in the words of Laplace,[1]

> "An intelligence knowing, at a given instant of time, all forces acting in nature, as well as the momentary position of all things of which the universe consists, would be able to comprehend the motions of the largest bodies of the world as well as the lightest atoms in one single formula, provided his intellect were sufficiently powerful to subject all data to analysis; to him nothing would be uncertain; both past and future would be present in his eyes."

As noted above, the liberating and galvanizing effect of these developments was, for good or ill, perhaps the decisive influence in creating the form and character of our present civilization; its consequences continue to unfold at the present time and will doubtless continue to do so throughout whatever future is left to us. To pursue these consequences as they have propagated through history is of consuming interest, but this is not my present concern. I wish to note here only two conceptual consequences of the Newtonian picture, which are most important to understand. These are:

1) On the basis of the Newtonian picture and its subsequent ramifications, we may say that we have come to have some understanding of the phenomena of inanimate nature. We have accomplished this without the postulation of any vital principle. But we must pay a heavy price for this, and the price is the following: *life now becomes something which itself must be explained.* As noted earlier, we always explain what we do not understand in terms of what we do; therefore life must be explained in the same way that the motion of the stars and planets was explained. Indeed, we are left no other possible mode of explanation of life than that exemplified by Newtonian analysis; nothing else can constitute explanation, almost by definition.

2) We have seen that the universality of the Newtonian world-view rests on the extent to which we can decompose any natural system into a set of material particles. The belief that any natural system can be so decomposed, and that the laws governing the motions of these particles can be determined, is the essence of *reductionism.* Reductionism thus serves to bring all natural systems under the purview of the Laplacian Spirit mentioned above. Concomitantly, the problem of actually analyzing a specific system to determine the character of the particles of which it is composed, and the nature of the forces acting on these particles, is an empirical problem, and not a conceptual or theoretical one. Thus, along with the growth of reductionism we find a corresponding rise in *empiricism.*

Empiricism initially is an extension of the commonsense view that in order to determine the properties of a system of interest, one must observe that system directly. The empirical approach dovetailed neatly with reductionism not only in the conceptual context we outlined above, but also in connection with technological developments; clearly one could only understand a

machine (e.g. a clock) by taking it apart, examining each of its constituents in detail, and determining how the operation of each constituent plays a particular role in the overall functioning of the machine. But the empiricist-reductionist view can be pursued independently, separated from the conceptual framework which gives it meaning; it then becomes fraught with dangers and pitfalls. For instance: measurement and observation are primarily ways for discriminating between systems; that is, they necessarily emphasize the idiosyncratic features of systems, rather than what makes them similar. Worse still, an exclusively empirical approach to natural systems has a way of degenerating into such things as positivism; the belief that the only qualities of systems which are real or meaningful are those which can be directly measured. Invariably such a belief is coupled with an extremely narrow view as to what in fact constitutes a measurement or observation, and leads inexorably to an assertion that whatever falls outside this narrow framework does not exist. Worst of all, an exclusively empirical approach gives rise to an utterly false dichotomy between "experiment" and "theory", which has done boundless mischief in the world, and about which we shall have much more to say subsequently.

Now the phenomena of biology, i.e. the behaviors exhibited by living systems, have always posed a critical challenge to both the Newtonian synthesis, and to the reductionist-empiricist approach to which it indirectly gave rise. This was clearly perceived by Einstein, who wrote once to Leo Szilard,[2] "One can best feel in dealing with living things how primitive physics still is...". We can sense the nature of the challenge of biology in that fact that, despite centuries of effort, no satisfactory definition of life or the living state has been forthcoming; no specific criterion or test which may be applied to a given system to effectively decide whether it is living or not. We all feel, I think, that there is some essential, directly perceivable distinction between the organic and the inorganic realms, but it has proved impossible to quantify this distinction. This failure is itself an important empirical fact, which as we shall see arises from a variety of circumstances. Prime among these, I believe, is that our perception of a system as living or not arises from a commonality it shares with other systems, and which is of a relational character. As noted earlier, physical measurements do not deal with such commonalities, but rather quite the reverse. Indeed, if we wanted to interrogate our Laplacian Spirit about the characteristics of a particular organism, he would find it impossible to give us an answer, or even to understand us; if all he knew were the positions and momenta of all the particles in the universe, he could never locate an organism in the particulate chaos, let alone tell us about it. From this it is clear that biology forces us to demand other kinds of information, and make other kinds of measurements, besides those with which physics is concerned. Moreover, these measurements must pertain to properties which classes of systems hold in common, and not to properties which simply allow us to distinguish one system from another.

The challenge of biology which we are discussing has been, if anything, intensified by several decades of explosive research in molecular biology, which represents the most recent attempt to enforce the reductionist-empiricist paradigm on biological systems. We have indeed learned a great deal about molecular constituents of particular organisms, but this knowledge has only served to widen the gap between what we know and what needs to be explained. The molecular biologist has asserted that all problems in biology have been "reduced" to physics and thereby solved; what has in fact happened is quite the reverse of this. Namely, by demonstrating the inability of conventional physical descriptions to deal effectively with the phenomena of that subclass of physical systems which happens to be organic, we thereby illuminate fundamental and heretofore unrecognized gaps in physics itself. In other words, far from biology losing its autonomy by disappearing into present-day physics, biology will necessarily force the physicist to fundamentally and radically revise the character and scope of his discipline.

All of this was clear to Ludwig von Bertalanffy more than forty years ago. He was perhaps the first to clearly articulate the various points we have made above; the fact that the study of living systems is a study of commonalities shared by systems of differing physical structures; that physics cannot encompass organic phenomena without fundamental modification and extension; and that the commonalities characteristic of biological systems are exemplary of other kinds of commonalities which could form the basis for a general theory of systems *per se*.

All this was manifested in von Bertalanffy's

analysis of the concept of *equifinality*, particularly as it appears in developmental biology. By equifinality is meant the stubborn tendency of a developing system to reach the same final state despite such experimental interventions as amputations, randomizations or hybridizations. Phenomena of this character are quite remarkable, and utterly different from the behavior of familiar physical systems; study of them led the embryologist Driesch to despair of ever finding a physical explanation for them, and thus to a thoroughgoing vitalism. Indeed, the phenomena of development in general have always been puzzling to physicists, who on the basis of the Second Law of Thermodynamics argue that material systems should proceed from ordered to disordered states; not the other way around. Von Bertalanffy was among the first to realize, and argue forcefully, that biological systems are open systems, able to exchange matter and energy with their environments, and consequently the Second Law is not even directly applicable to them; he was among the first to perceive the distinctions between the equilibria of closed systems and the steady states of open systems; he realised that the behavior of open systems in the vicinity of stable steady states exhibited all the properties traditionally associated with equifinality; and finally, on this basis, he could account for the apparently purposive, telic behavior of developing systems in an entirely plausible and satisfying way.

Now let us consider the revolutionary implications of this analysis.

First, von Bertalanffy explicitly showed that a basic characteristic of biological systems could be approached entirely through a functional or relational criterion (i.e. that the system be *open*) rather than through reductionistic analysis of the specific particles of which any particular system happens to be composed.

Second, von Bertalanffy was perhaps the first to stress the relation between regulatory phenomena in biology, particularly those which appear to involve purposiveness, and the formal concept of stability.

Third, von Bertalanffy concretely exhibited the incapacity of the physics of the time to cope with developmental phenomena such as equifinality. For instance, thermodynamics at that time was (and largely still is) restricted to closed, isolated systems very near to thermodynamic equilibrium. Clearly, such a thermodynamics is helpless in dealing with organisms. Within recent years we have indeed seen numerous attempts to develop a thermodynamics of open, far-from-equilibrium systems (the best known is probably the work of Prigogine and his school). It might be remarked parenthetically that these attempts, which seek to extrapolate from conventional closed-system thermodynamics, are in many ways quite unsatisfactory; it is likely that entirely new ideas, perhaps drawn from abstract approaches to stability (for example, from catastrophe theory) will be required. In any case, we see even here how biology forces physics into new directions.

Fourth, and perhaps most important, was von Bertalanffy's recognition of the fact that any open system, in the vicinity of an asymptotically stable steady state, must behave similarly to a developing organism exhibiting equifinality. To employ a graphic (if not completely satisfactory) term, such behavior is a *metaphor* for equifinality. Therefore, we see that there is a sense in which we can learn something about a particular system S, such as a developing organism, by studying some other system S′, perhaps completely different from S in any purely reductionistic or physical sense, but nonetheless manifesting the same *behavior*.

To me, this last proposition articulates the essential characteristic of system theory, which distinguishes it from everything that has gone before. Hitherto, the reductionistic-empirical approach to systems mandated that any given system of interest be analyzed into ultimate particulate units; understanding any behavior of the system meant resolving that behavior into the properties of these particulate units and the forces acting on them. On the other hand, von Bertalanffy stressed that in the organic realm, *many important classes of behaviors are shared by systems of the utmost physical diversity*. This fact has, in turn, two utterly revolutionary implications: first, that if such behaviors are to be analyzed, the appropriate analytic units cannot be the particles of the physicist, and hence, that the reductionist analytic paradigm is not the only tool available for the study of natural systems. Indeed, as we shall see, system theory shows that the reductionistic analytic paradigm is only one of the large family of analytic paradigms, each of which captures a distinct aspect of physical reality. The second implication, which arises from the first, is this: if two systems S and S′ which are physically different (i.e. are constructed of different kinds of particles) nevertheless behave similarly, then there is a sense in which we can

learn about S by studying S′. That is, *physically disparate systems can nevertheless be models, or analogs, of each other.*

To me, general system theory is precisely the study of the different analytic paradigms which can be applied to understand particular system behaviors, and the characterization of the circumstances in which a system S′ may be a model of a system S. All of this was clear to von Bertalanffy long ago, and these insights animated every aspect of his work.

There is one further aspect of these considerations which must be pointed out before proceeding further. I have already mentioned the fact that the Newtonian approach to the world already inextricably tied the study of natural phenomena to the properties of certain mathematical or formal representations of those phenomena. Precisely the same thing is true of the other kinds of analytic but non-reductionist paradigms which comprise general system theory. Indeed, in the last analysis, *two distinct systems S, S′ can behave similarly only to the extent that they comprise alternate realizations of a common mathematical or formal structure.* The study of this mathematical structure, through the techniques of mathematics itself, is simultaneously a study of all of the realizations of that structure; i.e. of an entire class of analogous but physically distinct systems. Consequently, general system theory also comprises, in addition to the various analytic paradigms applicable to natural system behaviors, the study of formal or mathematical systems as an essential part of the natural systems which realize them.

On the basis of what we have said so far, we can now begin to understand why the very existence of a general system theory is so profoundly alienating to many of those reared in the empiricist-reductionist tradition. It is also disturbing to those who feel that any form of analysis violates the integrity of a system. It is worth while taking some time to discuss these objections, which may with some justice be regarded as an autoimmune disease of the intellectual community, and which has been characterized by some bizarre and even sinister syndromes in the recent past, generally expressed in a variety of false dualisms: theory-experiment; holist-reductionist, etc., in which these terms are related as heresy is to faith, sin to virtue.

We have seen that the study of mathematical or formal systems, and the manner in which they can be realized in nature, is an essential part of system theory. There are many who simply reject out of hand the notion that there can be any relation between mathematics and the world of life, and hence who reject any study in which mathematical analysis plays a role. As the population biologist J. G. Skellam once wrote:[3]

> "Because of their extreme simplicity and abstract nature, mathematical models contrast so sharply with the intricate and colorful situations which they are intended to represent, that many biologists quite naturally have felt them to be fundamentally different in kind, and have therefore found them either inacceptable or worthless as aids to their work."

Indeed, there are certain intellectual spheres, notably those in which some kind of ineffable "inspiration" plays a primary role, in which the very term "mathematical" is the worst kind of opprobrium. This fact often takes amusing forms. Let us for example ponder some critical diatribes directed against a few well-known musical compositions:[4]

> "The First Symphony of Brahms seemed to us... mathematical music evolved with difficulty from an unimaginative brain... This noisy, ungraceful, confusing and unattractive example of dry pedantry... abounds in headwork without a glimmer of soul... we might pore over a difficult problem in mathematics until the same result is reached..."

> "(In) the Franck Symphony... one hears the creaking of machinery; one watches the development with no greater emotion than would be evoked by a mathematical demonstrator at a blackboard."

> "They (Reger's compositions) are like mathematical problems and solutions, sheer brainspun and unlyrical works."

> "In the Schoenberg *Variations*... one feels that Schoenberg has reached some such conclusions as "a straight line tangent to a circle is perpendicular to the radius drawn to the point of contact'."

> "The science of M. Berlioz is a sterile algebra."

> "The music of Wagner imposes mental tortures that only algebra has a right to inflict."

The poet Coleridge perceived an incredible antithesis between mathematics and imagination:[1]

> "I have often been surprised that Mathematics, the quintessence of truth, should have found admirers so few and so languid. Frequent considerations and minute scrutiny have at length unravelled the cause; viz. that though Reason is feasted, Imagination is starved, whilst Reason is luxuriating in its proper Paradise, Imagination is wearily travelling on a dreary desert."

It is clear from these few excerpts, and many others which might have been chosen, that mathematics is perceived by many as separated from the living world by an unbridgeable chasm. mathematics is mechanical, abstract, remote, inapplicable. The false dualisms implicit in these views are clear.

The above responses to mathematical investigations in science may seem comic. At least, they might simply be put down to individual idiosyncracy and dismissed. But at least twice within the past tragic half-century, they have taken much more serious forms. There is nothing funny in the arguments that mathematical investigations of natural systems are a pernicious debasement of science; or that they are incompatible with some state-supported orthodoxy, and hence indicative of disloyalty or treason to the State. For instance, the growth of National Socialism in Germany was accompanied by what was called "Deutsche Physik" or Aryan Physics. This was primarily a reaction on the part of certain experimental physicists against the explosive development of mathematical physics during the first few decades of this century (primarly relativity, but also quantum theory). It was argued that "those who studied equations instead of nature" were charlatans and worse; the true scientist could only be an experimentalist, who developed a kind of mystic rapport with nature through direct interaction with it. Likewise, in the Soviet Union, the growth of what came to be called Lysenkoism was closely linked with a phobia against mathematical investigation of biological systems, particularly in genetics. In both of these cases, we find a rebellion on the part of some empirical scientists who found themselves alienated by the new directions in which their science was moving, and who found it desirable to attack these developments by allying their viewpoint with a convenient ambient political and social philosophy. In both cases mathematical approaches were denounced as wicked; in Germany as materialistic, in Russia as idealistic; what was common here was the denunciation, and the state of mind which gave rise to it.

The other aspect of system theory which many empiricists find deeply disquieting revolves around the modelling relation between systems, which, as we have seen, implies that we can learn something important about a system of interest by studying a different system, which we may call a model system or analog. To many empiricists and experimentalists, who have devoted their lives to careful observation and experiment performed on a small number of specific systems, it seems somehow outrageous, or dishonest, to acquire information about these systems through general arguments; they regard it as akin to magic. Indeed, there is a sense in which the study of model systems does resemble the practice of sympathetic magic, and in which the creation of mathematical models resembles numerology. In the form of mysticism known as Kaballa, for example, specific numbers are associated with systems, through the fact that each system has a name, and the letters of the alphabet in which the name is written also have associated numerical values. Thus, a mathematical object (a number) is associated with a given system in a unique fashion. The basic hypothesis of Kaballa, the one which makes it mysticism rather than science, is that the name of a system is not arbitrary or accidental, but partakes of the reality of the system; thus particular properties of the system are manifested in corresponding numerical properties of the associated number, and can be studied thereby. The superficial resemblance of this procedure to mathematical modelling in science is clear; such modelling also proceeds by associating a mathematical object with a system, which we believe captures some aspect of the reality of the system; we seek then to learn about the system by studying the properties of the associated model.

The crucial difference between magic and science, as we have noted, resides in the *manner* in which models are generated, and through which specific properties of the modelled system are captured.

The animosities and misunderstandings aroused through system-theoretic modes of analysis, which as we have noted, revolve to one degree or another about the modelling relation between

systems, might lead one to believe that such relations are rare or obscure. Nothing could be further from the truth; in fact the modelling relation permeates our lives and experience, even when it is not ordinarily perceived as such. This being the case, it may be helpful to turn to some concrete examples, drawn from a broad spectrum of disciplines, to see how widespread these relations really are.

Let us begin with mathematics itself. We are all more or less familiar with the construction of mathematical representations or models of natural systems. But it is perhaps not widely recognized that mathematicians often construct mathematical models of other mathematical systems. Probably the best example of this activity is found in that branch of mathematics called *algebraic topology;* it is worth while spending a moment reviewing how this comes about.

As we all know, topology is that branch of mathematics which studies the notion of continuity. The systems studied in topology are thus those mathematical structures on which continuous processes can be defined; these are *topological spaces.* One of the basic problems of topology is to determine whether two such spaces are indistinguishable from one another from the standpoint of the continuous processes which may be defined on them; if this is the case, the two spaces are said to be *homeomorphic.*

Now topology in general is a very difficult subject, and the problem of deciding whether two spaces are homeomorphic is a very difficult problem. By contrast, the study of algebraic systems, such as groups or rings, is often very much simpler. It was Poincaré who first showed how it was possible to canonically associate algebraic structures (groups) with topological spaces, in such a way that homeomorphic topological spaces had associated with them isomorphic groups. These associated groups can therefore be regarded as *topological invariants,* and they precisely constitute algebraic models of the topological spaces from which they are constructed. The isomorphism of these groups, which can be relatively easily decided, then provides a necessary condition for the homeomorphism of the corresponding spaces. Further, specific numbers can be effectively associated with such groups in an invariant way (e.g. the rank of the group). These numbers are themselves topological invariants, in fact they play the same role for topological spaces that physical observables play for natural systems, and the formal machinery for computing them are the analogs of measuring instruments.

The field of algebraic topology is that portion of topology concerned with the construction and interpretation of algebraic models of topological spaces. The goal of algebraic topology is to construct a sufficiently broad spectrum of such algebraic models so as to completely classify topological spaces; this would in particular allow us to always decide when two given spaces are homeomorphic. Stated otherwise: each algebraic model captures some feature of the topology from which it is constructed; we seek a sufficiently comprehensive set of models so that *every* feature of the topology can be recaptured from them. It is still very much an open question whether this goal can be reached.

In any case, the relation between a topological space and an associated group is a modelling relation, and we see how we learn about the space by studying an entirely different system, the group. Now this relation between spaces and their associated groups is something which can itself be studied in a mathematical context. Such a study was initiated by the mathematicians Eilenberg and MacLane in a landmark paper entitled "The General Theory of Natural Equivalences" which was published in 1945. From this paper has grown a new branch of mathematics, which has come to be called the Theory of Categories, which cuts across all others. We can see, from what has been said so far, that Category Theory is in fact a general theory of the modelling relations which exist between formal systems. It is not surprising that this theory is coming to play an increasingly important role in general systems theory (which, as I have argued above, is precisely concerned with the establishment of modelling relations).

It is interesting to note that Category Theory has elicited exactly the same reaction within the community of pure mathematicians that system-theoretic approaches have generated in the community of experimental or empirical scientists. Initially, category theory was dismissed out of hand by many mathematicians as at best a cumbersome language for stating well-known facts about algebraic topology. Later, when it became clear that category theory could answer important open questions in a variety of areas, it was conceded to be of some limited usefulness for specific applications, but still of no significance as an independent branch of mathematics. This atti-

tude is still prevalent; indeed, just a few years ago, I served on the Executive Committee of the Department of Mathematics at SUNY Buffalo, when that department was in the process of hiring an eminent category theorist. A substantial group of senior faculty tried to block this appointment on the grounds that category theory was not real mathematics; they were dissuaded only with difficulty from writing to other departments to collect opinions on whether category theory was mathematics or not.

Before leaving this brief discussion of mathematics, we may note that the study of mathematical systems is one which cannot proceed according to a reductionist paradigm. We cannot learn much about a mathematical system, a group, say, be decomposing it into elements. Rather, we require higher-level analytic units, specifically related to the global properties which define the group. In general, I believe that we can learn much from the analogy between the mathematician's study of formal systems and the scientist's study of natural ones; the analogy itself may well be what is responsible for what Wigner called the *unreasonable* effectiveness of mathematics in the natural sciences.

Let us now turn to physics. As we saw earlier, the Newtonian world-view has both generated and validated the empirical, reductionistic approach which has dominated modern science for centuries. Thus it is perhaps ironic that one of the supreme achievements of physics should be of the non-reductionistic character typical of system-theoretic arguments. This is, of course, the mechano-optical analogy developed by Hamilton over one hundred years ago. Hamilton showed that two distinct and independent branches of physics, namely optics and mechanics, could be conceptually unified, not through any *reduction* of optics to mechanics or vice versa, but rather through the fact that both disciplines obeyed the same formal laws; i.e. realized the same formal system.

In more detail, it had been shown by Fermat that the facts of geometric optics could be understood by postulating that light will follow that path through an optical medium which minimizes (or more accurately, which extremises) the time of transit. Likewise, there was a long tradition, culminated by Hamilton himself, in which the Newtonian laws of motion of a material system were obtained by regarding the change of configuration of that system as proceeding along a path which minimized (or extremised) a mechanical quantity called *action*. Hamilton perceived that, on this basis, one could establish an exact analogy between geometric optics and particle mechanics; i.e. one could establish a dictionary by means of which any assertion in optics could be translated into an assertion about mechanics, and *vice versa*. Moreover, any behavior of a mechanical system could be *simulated* or *modelled* by constructing an appropriate optical system; conversely, any optical behavior could be simulated in some appropriate mechanical system. On this basis, Hamilton was led to what is now called the Hamilton–Jacobi partial differential equation for the spreading of wave-fronts of constant action, which is not only one of the crowning developments of 19th century science, but also provides the prototype for modern theories of optimal control. But this is not all. Hamilton was, of course, aware that geometric optics, governed by Fermat's Principle, was a limiting case of a more microscopic theory called wave optics. Hamilton himself established a one-to-one correspondence between particle mechanics and geometric optics. If he had had the least reason for doing so, he might have been tempted to argue that sitting underneath particle mechanics there was also a more microscopic theory, related to particle mechanics in the same way that wave optics is related to geometric optics. Such a more microscopic mechanical theory would naturally be called *wave mechanics*. Indeed, by exploitation of this analogy, Hamilton was fully capable of deriving Schrödinger's equation, nearly a century before it was actually derived by Schrödinger, who exploited precisely this argument. Such has been the power of analogy in physics, the bastion of reductionism.

In the past century, especially since the advent of relativity and modern quantum theory, physics has become more and more concerned with properties which remain invariant to certain classes of modifications of the system manifesting these properties. More generally, it has become concerned with expressing the manner in which physical quantities transform under such circumstances. These developments belong to a venerable tradition, whose earliest modern antecedents go back to Galileo. Among other things, Galileo investigated the manner in which the mechanical properties of structures change when the structures are subjected to a change in scale. He found, of course, that mechanical properties do

not scale linearly with geometric magnitudes in general, and that therefore structures which are *geometrically* similar will not in general be *mechanically* similar. Thus, if we wish to construct a full-sized building which is mechanically similar to a small model, we cannot simply scale up that model. These observations of Galileo provide the point of departure for the dynamical modelling which plays such a crucial role in our technology.

This kind of dynamical modelling provides a prime example of how we may study a structure of interest by investigating another system (which in this context is called a scale model). Typically, such modelling is our only recourse in system design, for we wish to determine the characteristics of a system which does not, as yet, even exist. We might hope to study these characteristics from first principles, employing physical laws, but as a practical matter this is often unfeasible. Therefore the employment of a scale model, as a kind of analog computer, becomes mandatory. Thus, our own technology provides abundant evidence of the significance of modelling relations as an alternative to the reductionistic paradigm.

All these considerations lead us naturally back to biology. The kinds of scaling laws just mentioned apply as much to organisms as to other kinds of physical systems, and there is a long tradition of investigation of biological similarities, particularly in connection with growth and with evolution. It might be mentioned parenthetically that von Bertalanffy was, of course, much interested in these developments, and himself proposed a law of growth for a class of biological systems which now bears his name. More generally, the study of quantities which remain invariant while the underlying system is subjected to transformations (e.g. is growing or evolving), which as we have seen is characteristic of much of modern physics, is also the province of what in biology is called homeostasis. In this way, we come back full circle to the phenomena with which we started, for homeostasis is simply another language for describing equifinality, stability, and control.

Biology is itself replete with examples of analogs and model systems. Many of these, oddly enough, are widely employed in empirical studies. For instance, many enzymologists, interested in studying the catalytic mechanisms whereby enzymes function, often study completely inorganic systems (such as colloidal metal suspensions), which they terms *enzyme models.* Membranologists, interested in the permeability mechanisms of biological membranes, construct and study a bewildering variety of films and ion exchangers which are collectively called *model membranes.* There is an extensive literature on so-called *neuromimes;* synthetic systems which can generate and propagate an action potential, and hence exhibit excitability. As we abundantly pointed out previously, the relation between such model systems and their biological prototypes cannot reside in any identity at the particulate level, but must reside in common functional or relational properties which are preserved invariant when we replace the prototype by the model. Thus, the wide employment of model systems already reveals the tacit recognition of analytic alternatives to reductionism.

A more startling fact about biological systems is that profound analogies are found between different levels of biological activities themselves. These are of the same character as the mechano-optical analogy between different classes of physical systems, which we have already discussed above; some of the biological analogies may ultimately turn out to be as significant. Let us mention just one: the analogy between the neural networks of higher organisms and the genetic networks of individual cells.

Neural networks have, of course, played a fundamental and many-sided role in the history of system theory. They were initially introduced by McCulloch and Pitts in 1943, as discrete, Boolean versions of the continuous-time, dynamical networks introduced earlier by Rashevsky and his collaborators. Technically, the passage to discrete time served to sidestep the then insuperable analytical problems arising from the systems of non-linear differential equations which characterize the continuous-time case; the study of neural nets requires only algebra and combinatorics. It quickly became clear that the neural networks not only furnished key insights into the operation of the brain, but also indicated how one might fabricate new systems which could behave in a brainlike fashion. Consequently, neural networks became one of the primary motivations of the theory of automata in all of its ramifications, especially in computation.

The theory of genetic networks, on the other hand, arose from an attempt to come to grips with the fact that cells with the same genes could nevertheless be very different from one another.

This is obvious in the differentiated cells of a multicellular organism, which in fact constitute a clone; but it is also true in bacteria. Indeed, many bacteria can adaptively modify their chemical machinery very quickly in the face of varying environmental conditions. It was clear from such facts that merely specifying a cell's genome is not sufficient; superimposed on the genome must be some kind of control mechanism which will determine whether, and when, any specific gene will be expressed (i.e. turned on).

On the basis of their experiments with bacterial systems, which were then blithely extrapolated to the rest of the biosphere, the French microbiologists Jacob and Monod proposed such a control mechanism. In brief, these workers proposed the existence of a new formal unit of control, which they termed an *operon*. A genome was conceived as a set of these formal units. More than this: the operons in a genome were in constant communication; the vehicles for this communication were postulated to be chemical specificities, residing in binding sites in the control region of the operon. Such specific interactions serve to turn the operons on and off, depending on initial conditions and environmental circumstances, with concomitant expression of the corresponding genes.

The analogy between a network of operons and a network of neurons is clear. Indeed, some of the simple operon networks which appear in the original papers of Jacob and Monod, illustrating how various kinds of differentiation can take place in cells, are formally identical with neural networks proposed a decade earlier in relation to such mental phenomena as learning and conditioning. Some rich rewards have already been gained from exploitation of the analogy, but certainly many more still await discovery.

Several features of the operon system are worth pointing out, in connection with our earlier discussion of alternatives to the reductionist paradigm in biology. For instance: it must be stressed that the operon is a purely formal unit. It is not a physical particle, nor is it composed of such particles. It cannot be isolated physically from the cell to which it belongs. It is thus not like a neuron in a neural net, which we may at least hope to identify with a discrete anatomical structure; namely, a biological nerve cell. In short: the operon is a purely abstract unit of analysis, which cuts obliquely across the particulate structures provided by the reductionistic paradigm. It is, in fact, precisely the sort of analytic unit which we argued is required in dealing with functional activities manifested by large classes of similar organic systems.

The communication channels which turn a set of operons into a network are likewise not physical structures, as are the axons in the brain. They are again purely formal units of analysis, whose properties ultimately rest on specific active sites at the molecular level. These sites themselves also constitute examples of non-particulate functional units; they cannot be physically extracted from the molecules which bear them, nor can they be characterized in any simple fashion in terms of quantum-theoretic descriptions of those molecules. They in fact constitute a new and separate class of microphysical systems; a set of specific functional variates which are linked together, but which are not physically separable into a specific particle. Such considerations indicate yet another way in which biological activities force reconsideration of physical descriptions, this time at the most fundamental microscopic levels.

The point of these various examples has been to exhibit the ubiquity of modelling relations between systems of different physical natures; to show how they have been (and can be) used to allow us to understand the properties of physically distinct systems in a unified context, and to point out how the existence of a modelling relation between systems indicates the further existence of an underlying mathematical structure which the given systems realize. We have also seen how the modelling relation forces us to recognize other sets of analytic units besides the particles provided in the reductionistic paradigm. Such units are no less physically real than the particles, and enter into explanation of natural phenomena on the same footing. It is thus meaningless to ask which mode of analysis is "primary" in some abstract sense. Indeed, it generally happens that several different analytic modes may be brought to bear on the same system. Indeed, I would define a complex system to be one which may be analyzed in more than one way. When this happens, it becomes important to establish relations between the analytic units involved; but in general this relation is not a reductionistic one.

Let me conclude this survey of system theory with a brief discussion of one more way in which the modelling relation is intertwined with the

study of life; one which I think will be most important in the years to come. Specifically, I would like to discuss the concept of an *anticipatory system.*

As was noted previously, von Bertalanffy was the pioneer in showing how apparently purposive behavior in biological systems could be understood in terms of stability. We also saw how such ideas are related to the concept of homeostasis, and thence to the crucial idea of feedback. Now the essence of feedback control is that it is *error-actuated;* in other words, the stimulus to corrective action is the discrepancy between the system's actual present state and the state the system should be in. Stated otherwise, a feedback control system must already be departing from its nominal behavior before control begins to be exercised.

On the other hand, we know from introspection that many, if not most, of our own conscious activities are generated in quite a different way. We typically decide what to do *now* in terms of what we perceive will be the effects of our action at some *later* time. To take a simple example: if we are walking in the woods, and see a bear appear from behind a tree, we need not wait until he is upon us before vacating the area. Rather we leave immediately, because we can *anticipate* the unpleasant consequences of continuing our walk. The vehicle by which we anticipate is in fact a *model,* which enables us to pull the future into the present. The stimulus for our action is in fact not simply the sight of the bear; it is the prediction or output of our model under these conditions.

This simple example contains the essence of anticipatory behavior. It involves the concept of *feedforward,* rather than feedback. The distinction between feedforward and feedback is important, and is as follows. In a feedback system, as we have seen, control is error-actuated, and serves to correct a system performance which is already deteriorating. In a feedforward system, on the other hand, system behavior is *preset,* according to some model relating present inputs to their predicted outcomes.

The essence of a feedforward system, then, is that present change of state is determined by an anticipated future state, computed in accordance with some internal model of the world.

Despite the ubiquity of feedforward modes of control, at all biological levels, anticipatory behavior of this kind has hardly begun to be seriously studied. There are several reasons for this. First, feedforward behavior seems frankly telic, or goal directed. The goal is in fact built in as part of the model which transduces between predicted future states and present changes of state. But the very suggestion that a behavior is goal-directed is repellent to many scientists, who regard it as a violation of the Newtonian paradigm.

Secondly, anticipatory systems have always been rejected out of hand in formal approaches to system theory, because they appear to violate causality. We have always been taught that we must not allow present changes of state to depend on future states; the future cannot affect the present. But in fact the entire purpose of a dynamic model is precisely to pull the future into the present; if we have a good model, and act by virtue of its predictions, we thereby approximate to a true anticipatory system.

If indeed feedforward or anticipatory control is as ubiquitous as it seems to be, then a number of fundamental new questions are posed to us. Among them are the following. Can we truly say we understand the behavior of such a system if we do not know the model employed by the system? How is it possible to determine the character of that model, in terms of measurements or observations performed on the system? More generally, under what circumstances is it possible for a system to contain an internal model of its world? What relations must exist between a set of indicators (environmental signals) and system effectors which will allow an effective feedforward control model to be constructed? How can the behaviors of different systems, perceiving the same set of circumstances but equipped with different models, be integrated? (This last is essentially the problem of conflict and conflict resolution.)

The questions just raised bear directly on the present search for forecasting and planning technologies to guide our behavior in the political, social and economic realms. Tacit in this search is the perception that our society and its institutions can no longer function effectively in a cybernetic or reactive mode; it must somehow be transformed into a predictive or anticipatory mode. That is, it must become more like an organism, and less like a machine. Such problems fall directly within the purview of system theory, which as I have repeatedly emphasized, is concerned with modelling relations between systems

in all their aspects. It is with such questions that I believe system theory will become increasingly involved in the coming decades.

In dealing with these problems, I reiterate my conviction that the properties of biological systems will provide crucial insights. Indeed, considered in an evolutionary context, biology represents a vast encyclopedia of how to effectively solve complex problems; and also of how not to solve them. Biology provides us with existence proofs, and specific examples, of co-operative rather than competitive activities on the part of large and diverse populations. These insights represent natural resources to be harvested; resources perhaps even more important to our ultimate survival than the more tangible biological resources of food and energy. But to reap such a harvest, we need to fabricate the proper tools. I believe we are indeed learning how to forge these tools, and that in these treat endeavors, the concepts of system theory pioneered by von Bertalanffy and others like him will play a pre-eminent role. We are faced with great challenges in the years to come; if we meet them with wisdom, these challenges will enable us to learn and grow. Ludwig von Bertalanffy was a wise man, and a noble man; if we can assimilate his insights, and continue making progress along the paths he illuminated, we will not go wrong.

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A COMMENT ON STRUCTURAL STABILITY

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It is argued, on the basis of some simple examples, that the concept of structural stability, as a scientific principle, is difficult to relate to *a priori* mathematical characterizations in terms of genericity or transversality. In general, the employment of mathematical objects to represent physical situations carries with it a number of tacit constraints, which impose crucial biases on the mathematics. This is illustrated by the disparity in behavior between pure real and pure imaginary eigenvalues of simple linear systems.

INDEX TERMS: Structural stability, linear systems, genericity, yransversality.

The publication of Rene Thom's Catastrophe Theory in 1972[1] has generated a widespread interest in the notion of structural stability. Indeed, Thom has identified this mathematical concept closely with the intuitive idea of *form*; whatever else form may be, it must have the property of being impervious to small perturbations. This resistance to perturbations, when precisely formulated, is the essence of structural stability.

Thom's theory proceeds by further identifying structural stability with the geometric concept of *transversality*. In the simplest case, two curves in a plane meet transversally at a point if they do not share a common tangent at that point (hence two curves which do not intersect at all are transversal). Transversality exhibits the basic character of structural stability; if two curves meet transversally, they will continue to do so if subjected to a small perturbation. Conversely, if two curves do not meet transversally, then an *arbitrary* small perturbation will generally produce transversality.

From this it is a small step to the argument that transversality is somehow common, or *generic*, while its absence is correspondingly rare, or non-generic. If we were to talk about transversality in probabilistic terms, we would say that the set of non-transversally intersecting pairs of curves in a plane form a "set of measure zero" in some appropriate measure.

Thom's basic Classification Theorem flows from a relentless application of these simple ideas, and from the argument that, should transversality fail, that failure will itself occur in a generic or structurally stable fashion.

We have noted elsewhere[2] that ideas of this sort must be handled with a certain caution when extrapolated from mathematics to physics and biology. Basically, this is because it is hard to decide *a priori* what is probable and what is improbable. The main vehicle for making such decisions has been the familiar Lebesgue measure, through the identification of probability and measure theory by Kolmogorov in the 1930's. Lebesgue measure is certainly the "natural" measure of geometry. But let us cite two caveats to its unrestricted extrapolation to physical and biological arguments:

1) On the real line, the Lebesgue measure of the set of rational numbers is zero. This means that it is "infinitely unlikely" that a number chosen from the reals will be rational; or, stated another way, that to be irrational is generic. On the other hand, if someone is asked to "pick a number", he will with overwhelming probability pick a rational number. This curious circumstance arises because human beings are apparently not equipped to approach the real numbers in an unbiased way. Hence, in such a circumstance the direct identification of "genericity" with the abstract properties of Lebesgue measure is certainly not warranted.

2) In a long series of papers and books,[3,4] Walter Elsasser has argued that the classical techniques of statistical physics, which are based entirely on Lebesgue measure, are in principle inadequate in confronting basic questions of biological organization and behavior. Roughly, this is because the set of physical states of an organism which are compatible with life form a subset of vanishingly small measure in the set of

all possible states. Thus as far as biology is concerned, everything of interest occurs on a set of measure zero, and hence disappears in the course of any statistical argument. This circumstance leads Elsasser to argue that the properties of organisms ate governed by laws ("biotonic laws") compatible with physics but not in principle inferable from physics.

In the present note, we wish to show that even in purely mathematical terms, the ideas of transversality and structural stability can confront similar biases which profoundly affect genericity. Let us consider the simplest kind of dynamical system; a linear system in the plane, whose equations of motion can be written as

$$\begin{cases} dx/dt = ax + by \\ dy/dt = cx + dy. \end{cases} \tag{1}$$

The eigenvalues of this system are given by

$$\lambda_{1,2} = \frac{(a+b) \pm \sqrt{(a+d)^2 \pm 4(ad-bc)}}{2} \tag{2}$$

It is well known (cf. Rosen, 1970[5]) that aside from the case of zero eigenvalues, which we may exclude from the discussion, the *only* structurally unstable situation arises when the eigenvalues of (1) are pure imaginary. The phase portrait in this case constitutes a one-parameter family of concentric ellipses about the neutrally stable steady state at the origin. An arbitrarily small perturbation of the coefficients (a, b, c, d) in this case will generally transform this steady state to a stable or unstable focus, and the trajectories to spirals into or away from this focus.

On the other hand, if the eigenvalues of (1) are real, the origin is a stable or unstable node, and this is a structurally stable situation; a small perturbation of the coefficients in this case will not alter the stability of the origin, nor the qualitative character of the trajectories.

Thus the question arises: from whence comes this difference between the real axis and the imaginary axis in the complex plane? On abstract grounds, one of these lines should be just as generic (or non-generic) as the other. Yet when employed as the basis for establishing the structural stability of the system (1), we see a vast difference between them; it is infinitely easy to force the eigenvalues of (1) off the imaginary axis by an arbitrary perturbation of coefficients, but it is actually *impossible* to force these eigenvalues off the real axis by such a perturbation.

The algebraic reason for this vast difference between the real line and the imaginary line can readily be seen from (2). In order for the eigenvalues of (1) to be real, it is only necessary that an *inequality* be satisfied; namely

$$(a+d)^2 > 4(ad-bc).$$

If the coefficients (a, b, c, d) initially satisfy this inequality, then they will continue to do so after an arbitrary but sufficiently small perturbation. On the other hand, if the eigenvalues of (1) are to be pure imaginary, in addition to a similar inequality, it is also necessary that an *equality* be satisfied; namely

$$a+d=0.$$

This equality does not survive arbitrary small pertubations of a and d.

Thus we see that there is an inbuilt bias towards the stability of real-eigenvalue systems, just as there is an inbuilt bias towards the rational numbers in us, as noted above. This is true even though there is no intrinsic *geometric* difference between the real axis and the imaginary axis of the complex plane, and that it would seem *a priori* just as easy to perturb eigenvalues off one of them as the other.

The ultimate source for the bias away from imaginary eigenvalues lies in our predilection for real coefficients in (1). Making the coefficients complex restores the equivalence of the two axes as far as structural stability is concerned. However, the employment of dynamical systems like (1) as models of physical or biological situations, in which rate constants like (a, b, c, d) tend to be assigned real values, creates a corresponding bias in behavior which must be taken into account. These considerations reinforce the comments made above, that the ultimate scientific validity of *a priori* concepts like transversality depends on how they survive the biases imposed by what we may call real-world constraints. We may note that, curiously enough, the study of the relation between genericity and the imposition of such constraints may itself fall under the rubric of catastrophe theory.

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For photograph of Dr. Rosen, please see Volume 1, *No.* 1, *p.* 66 *of this journal.*

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THE SOCIETY FOR COMPUTER SIMULATION
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Book Reviews

The editor invites books for review on any subject relevant to general systems. Books for review must be sent to Dr. George Klir, Editor, International Journal of General Systems, *Department of Systems Science, School of Advanced Technology, State University of New York, Binghamton, New York* 13901, *U.S.A. (and* not *to the publisher).*

FUNDAMENTALS OF MEASUREMENT AND REPRESENTATION OF NATURAL SYSTEMS, by Robert Rosen, North-Holland, New York, 1978, 221 pp.

This is an interesting book, but a difficult one to review. It is a highly personal book, written from a very personal perspective. As such, it had for me much of the excitement and stimulation of a very long conversation with someone who has thought deeply about the difficult problem of representation and modeling of real-world systems. As is true with many such conversations, I found it most stimulating when disagreeing with the author or objecting to the presentation.

After a chapter which gives some requisite mathematical background (a good introduction to the topic of equivalence relations), Rosen proceeds to develop a formalism within which he can address the problems of measurement of natural systems, of recognition, of discrimination between systems and of classification of systems, all of which can be regarded as part of the representation and modeling problem. In this development, he defines a number of important concepts and poses a variety of thought-provoking questions. This is value enough for a single book, even if one may disagree with some of the conclusions or even object to the manner of arriving at them. Some of the particularly interesting discussions involve the definition of *meter* and its relation to the concept of *observable*, the notion of linkage between variables, the idea of structural stability as an appropriate generalization for the concept of *dimension*, and the relation between the concept of model and that of *subsystem*.

The general subject of symmetry, scaling and equivalence (principally Chapter Six) is treated in an interesting and suggestive way, which demonstrates the importance of these concepts to any theory of modeling of real-world systems. The potential usefulness of the broader framework of category theory is indicated, a point which Rosen has advanced for some time.

In a number of ways, however, Rosen invites criticism or at least disagreement. Foremost, because of its central role, is the way he treats the relation between the concept of *state* and of *observable*. He appears to take the concept of state as primitive to the definition of observable. Many authors consider it more philosophically satisfying and empirically more useful to adopt the view of *state* as a conceptual construct which serves to parameterize the space of observable input-output pairs. It should be noted that in quantum mechanics, which Rosen appears to draw from, the state description was developed in just such a way.

In many of his arguments, he varies from the painstakingly formal to the facile, failing to distinguish between that which "seems" so or is "strongly suggested" and that which is well established. An example is provided by Section 5.5 in which he argues from his own definition of reductionism and a simple "test" of the reductionist argument to the firm conclusion that reductionism is of value only where it is superfluous. The conclusion may be valid, but his argument does not establish its validity. Section 5.4, leading to dismissal of stochastic approaches, is in my view comparably flawed.

Especially regrettable is Rosen's failure to demonstrate the empirical content of the formalism; I had great difficulty seeing the empirical potential of many of his arguments. In some cases the empirical content seemed rich and in other cases, I came away convinced that it was nonexistent. Empirical examples would have helped. However, the few examples included were not always well developed. The example in the introduction relating cellular concentration of chemical metabolites to cell biochemistry neglects cell structure and the localization of concentrations. Another

example involves genetics and natural selection. Here, I find his formulation interesting and suggestive, but hardly compelling. Scant attention is given to the relation between theory and observation; he eschews the reductionist viewpoint, but offers no alternative bridge to reality. He bases a detailed argument on a posited relation between environment and phenotype, which is justified simply by prefacing it with the word "Intuitively" (to me, it is intuitively *not* so). Moreover, he appears to ignore all of the mechanistic knowledge of genetic transmission that has even been developed.

It is, as I have said, a very personal book. Rosen warns us that this is so in the preface. I am sorry that he has chosen to ignore virtually all of the recent work on representation of natural systems: that of Zeigler, Mesarovic, Zadeh, Klir, Laszo, Rapoport, Weinberg, Varela, and many others.

I found the book interesting and stimulating for the questions it raises and the directions it points in terms of seeking answers. It certainly led me to a deeper understanding of the subject. It is a book that many students of modelling and representation will want to read. However, it is a heavily biased treatment and must certainly be balanced by the study of other authors.

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AN INTRODUCTION TO MATHEMATICAL MODELLING, by E. A. Bender. Wiley-Interscience, New York, 1978, xvi + 448 pp.
MODELLING OF COMPLEX SYSTEMS, AN INTRODUCTION, by V. Vemuri. Academic Press, New York, 1978, x + 256 pp.

We are beginning to see an outwelling of textbooks at the senior/freshman level on modelling. The emergence of this phenomenon is due to a number of factors including: the recognition by mathematics departments that the jobs for their graduates lie in real world application; the ever increasing use of models in the various disciplines and especially the interdisciplines; and the expanding role of the computer in science and industry.

Under review are two recent candidates, both quite acceptable as course texts, but differing interestingly in approach. Bender's book is intended for beginning applied mathematics students. Vemuri's is aimed at an interdisciplinary audience most likely to be found in systems science or systems engineering programs. While basically dealing with the same material, the author's styles, flavors and emphases are different which reflect the biases of two main root sources of the emergent discipline.

The most striking contrast in approaches is the attitude taken to methodology. Bender eschews discussion of methods in favor of examples. His book thus largely consists of a compendium of modelling examples partitioned according to some methodological similarity criteria (e.g., arguments from scale, graphical methods, basic optimization). The idea, presumably, is that students will infer the methods by intimate ("learning by doing") exposure to the examples. While we may agree with his statement that "purely mechanical approaches usually end in failure" we need not jump to his apparent conclusion that explicit presentation of methods, where they exist, has no advantages (or is even harmful?). There are traces of the mathematician's aesthetics here: the imaginative leap, the novel formulation, the clever proof. Yet this cannot be the whole story in modelling. Witness Bender's outline of a modelling procedure (1. Formulate the problem, 2. Outline the model, 3. Is it useful? 4. Test the model) given with the predictable caveat ("giving rules for [model building] is like listing rules for being an artist"). Despite the disclaimer, it is to the book's credit that this outline is quite often followed in the treatment of many of its models.

In contrast to Bender's, Vemuri's approach is thoroughly methodologically oriented. Chapters are devoted to a survey of large scale systems, the principles of systems theory and the modelling process. Such concepts as measurement scales, algorithms, parameter identification, cluster recognition, linear programming and linear systems are discussed in methodological terms: what are the objectives, what methods are available, where do they apply, what are the structures employed, etc. In most cases, concepts and methods are sufficiently illustrated by examples and exercises. Unfortunately, this is not the case for the treatment of modelling and simulation. The high level discussion is excellent but it remains too abstract to be grasped by the novice. This is perhaps the best that can be done at the present time in the case of modelling, but simulation languages are by now sufficiently understood that they can be

treated to the same depth as linear programming. In other words, if a reasonable outcome of the course is an elementary ability to do linear programming, then an equally reasonable outcome is an elementary ability to do simulation programming, especially given the title of the book.

In terms of readability, Vemuri's book is easy to read and with the exceptions already noted, can probably be understood by a student on his own. Bender's style on the other hand is terse (his book is half the size of the other) which I found difficult going. It is true that this demands more participation from the reader. Often, however, this may be more frustrating than enlightening (especially if key concepts are omitted, as in the treatment of dimensional analysis, where the definition of terms in equations is not given thus rendering Buckingham's Theorem meaningless and its proof wrong). The book, on the other hand, has a wealth of interesting and unusual examples to serve as seeds for interpretation and exploration. But to teach from the book, an instructor must be willing to do some preliminary interpretation and source reading himself.

Both books would be valuable on a modelling instructor's bookshelf. Bender's for its variety of models (nicely indexed by topic), Vemuri's for its systematics, both for their entrées to the dispersed literatures. Neither can claim to be the last word in teaching introductory modelling methodology.

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RECOGNITION OF PATTERNS USING THE FREQUENCIES OF OCCURRENCE OF BINARY WORDS, by Peter W. Becker. Springer Verlag, New York, 1978 (3rd revised and enlarged edition).

It is a pleasure to welcome the third revised edition of the book by Dr. Peter W. Becker, which first appeared in 1968[1] as a subsequent version of the author's doctoral dissertation and which is now promoted by an international publisher.

This work should be considered as a thorough self-contained monograph of pattern recognition methodology based on sequences of binary words. It also gives a detailed exposition of the FOBW procedure, which uses the "frequency of occurrence of binary words" as features for classification among a few possible classes. It includes an efficient heuristic procedure by which longer well-separated N-grams are generated from shorter inadequate ones. Using excellent examples and applications, empirical evidence to the quality of these procedures is given; the author usually relies on conjectures, of which some may have been proven later by other researchers, i.e., in the case of his own S-measure of separability. The FOBW method clearly has limitations, but in our opinion it is probably very useful in those circumstances where the signals are nonstationary, the number of classes is limited by four to five, and time-normalization is unfeasible.

The book has been divided into eight chapters. The opening Chapter 1 is a survey of classical pattern classification approaches and applications and makes the first part of another book.[2]

Chapter 2 describes the FOBW procedure step-by-step and is recommended to be read first by specialists. In Chapter 3, dependency is examined among all the N-gram and delayed N-gram frequencies, which leads to a set of independent features. Chapter 4 discusses the S-measure of separability, which is the ratio of the difference of the means to the sum of the standard deviations of the class-conditional density functions. In Chapter 5, the FOBW values are related to the Markovian synthesis of sequences of binary words. Chapter 6 contains the above-mentioned original procedure for N-gram lengthening and conditioning. Lastly, Chapter 7 describes two hardware implementations for jet engine sounds and isolated word recognition; and Chapter 8 is a summary.

An extensive bibliography with over 200 references widely covers the pattern recognition research literature from the mid 1950's to 1970; unfortunately, this list has only been supplemented by very few recent publications up to 1977 as a list of some recent books. Most of them, however, are interesting from an engineering point of view. *No subject index is included.* The publication of this third edition is clear evidence of success. When compared to the previous edition, it contains a new list of errata, and one appendice dealings with proofs of some important results on multivariate distributions with specified marginals.

In conclusion, this book is recommended as a reference to all signal-processing or pattern re-

cognition specialists interested in binary-coded waveforms. The notation is often uncommon, but a careful study will be largely rewarded by many valuable ideas in relation to hardware design and signal classification.

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REFERENCES

1. P. W. Becker, *Recognition of Patterns*, Polyteknisk Forlag. Copenhagen, 1968.
2. P. W. Becker, *An Introduction to the Design of Pattern Recognition Devices*. Springer-Verlag, New York, 1972.

DYNAMICAL SYSTEMS AND THEIR APPLICATIONS: LINEAR THEORY, by John L. Casti. Academic Press, New York, 1977, 240 pp.

The number of books on system theory can be considered remarkable compared with the time that has elapsed since the emergence of this science. While some of the published books are real cornerstones in the foundations of system theory, others are of more limited interest; every new book in this field finds, however, many touchstones on his way.

The recent book by Casti is a useful and welcome addition to existing ones. This book covers the topic of linear dynamical systems and is structured into eight chapters.

The first chapter gives a general panorama of the problems which are more deeply treated in the remaining parts of the book and introduces fundamental concepts regarding external and internal descriptions for dynamical processes, realizations, controllability and observability, stability and feedback, optimality and control in the presence of disturbances. Many specific examples clarify and concretize the above theoretical concepts. These examples, in contrast with almost all previous books, do not regard the usual electrical or mechanical systems but cover such different fields as industrial production, water reservoir dynamics, economic planning, population migration, predator-prey systems and storm-sewer control. Nine references complete Chapter 1.

Chapter Two deals with the mathematical description of linear dynamical systems. Emphasis is given to a rigorous definition for dynamical systems based on the concept of internal state. External descriptions are then defined, together with frequency domain analysis and impulse response functions. Ten exercises and nine references complete the chapter.

The controllability and reachability properties of dynamical processes are analyzed in Chapter Three for time-dependent and time-invariant continuous and discrete-time systems. Remarkable examples involving macroeconomics and national settlement planning are given to explain these concepts. The first part of this chapter can be considered as classical. The subsequent sections of the chapter treat the problems of control with positive inputs and of control to given subspaces. The links between controllability and structural properties are then investigated and systems with delays considered. Further examples consider VTOL-type vehicles and urban traffic flow. The chapter includes twenty-three exercises and twenty-three references.

Chapter Four reports on the analysis of the observability and constructability properties of dynamical systems with examples from pharmacokinetics, satellites, input/output economics and air pollution. Ten exercises and twelve references are reported.

Chapter Five deals with a very important area of modern system theory, i.e. the invariant characterization of dynamical systems under a given group of transformations. The structural insight of this approach has only recently been fully recognized and is adequately treated here. Chapter Five includes one example, twenty exercises and sixteen references.

Chapter Six deals with realization theory and considers the questions of albegraic equivalence and minimal realizability, the construction of (minimal) realizations, the realization of transfer functions and the partial realization problem. Five examples, nineteen exercises and sixteen references can be found in this chapter.

Chapter Seven is concerned with stability theory and, after the basic concepts, the classical Routh–Hurwicz and Lyapunov methods as well as frequency-domain techniques are given. The stability of feedback systems, modal control, stabilization by means of feedback from observers and structural stability are then treated. This chapter is completed by three examples, twenty-one exercises and twenty-four references.

The last chapter treats the linear-quadratic-gaussian problem. Motivation and examples are considered first. Open-loop and feedback so-

utions are then compared and optimality versus stability conditions investigated. Subsequently, the solution of the Riccati equation, the structural stability of optimal closed-loop systems and some elements of linear filtering theory are treated. In this chapter remarkable examples like the water quality control and the armament race between competing nations are used as examples. Thirty-nine exercises and forty-nine references are also given.

After having read this book it is not easy to perform a comparison with existing volumes; it represents, in my opinion, a modern and very effective attempt to present linear system theory. The qualifying attributes of the book consist in:

(1) The exceptional interest and variety of real-world examples, which fulfill the double purpose of giving great clearness to the physical meaning of many fundamental concepts and of showing the power of the unifying approach offered by systems analysis;

(2) The completeness of the treated material does not limit the clearness of the exposition since the emphasis is always on the concepts and not on the technicalities. State-of-the-art themes are included and neatly treated;

(3) The subdivision of the arguments in the different chapters is very suitable for educational purposes.

At the end of the preface the author states that "A student who masters the material of this volume will be well prepared to begin serious system-theoretic work either in the applications are or in graduate research in systems-related disciplines". This is certainly true; this book can be a valuable tool for students and deserves the interest of researchers in the wide system theory field.

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TIMS MANAGEMENT SCIENCE

Volume 23, Number 8 April 1977

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100876.1 and 100876.2
FOUNDATIONS FOR A SYSTEMATIC PROCESS OF ORGANIZATION STRUCTURE DESIGN: PART I, THE ALGEBRAIC SPACE; PART II, SOME PROPERTIES OF ORGANIZATION STRUCTURES, by Helmy H. Balign and William E. Damon, *Graduate School of Business Administration, Duke University, Durham, North Carolina.* Part I (#100876.1), 30 pp. ($3.00). Part II (#100876.2), 36 pp. ($3.50).

In two companion papers the foundations for a systematic process of organization design are given. An algebraic space of organization structures is developed in Part I, and is used to identify an operational and efficient process of organization structure design. In the companion paper (Part II), various properties defined over the space and likely to determine the returns and costs of various organization structures are developed. The properties are given in the operational terms of the algebra, and are further investigated in those terms. These operationally defined properties, e.g., centralization, may then be used to make the process of design even more efficient by identifying desirable properties to be used to guide the process. The whole issue of developing a systematic process of organization structure design begins with the creation of an algebraic space of structures.

101077.1
INVENTORY OF SYSTEMS RESEARCH IN THE NETHERLANDS, by Johannes van der Zouwen, *Vrije Universiteit, Vakgroep Methoden en Technieken, De Boelelaan* 1115, *Amsterdam-Btv., The Netherlands*, 15 pp. ($1.00).

An inventory contains a name list of more than 70 researchers and/or research institutions with their addresses and research areas in brief description. The inventory is an English version of the original "Inventarisatie van het Systeem-onderzoek in Nederlan" which was published in the 47th "Nieuwsbrief" of the systeemgroep Nederland in Dutch.

110278.1
A DUAL MODEL OF THE BRAIN, by John H. Andreae, Ronald B. Dowd, *Department of Electrical Engineering, University of Canterbury, Christchurch, New Zealand* and Olive J. Webb, *Department of Psychology, Sunnyside Hospital, Christchurch, New Zealand*, 36 pp. ($3.50).

Most modelling of systems is done on the conventional, serial, digital computer. There are no other effective means available and there are no signs of a new simulating machine "round the corner". Models of general use must be designed for the digital computer.

The brain is different things to different people. Models of the brain are of interest and of use to many people: psychologists, physiologists, neurologists, biochemists and many others. It will be argued that, because of this diversity of interests, a comprehensive model of the brain must be at least two models: a fast behavioural model suited to serial computation and a slow structural model emphasising the parallel operations within the brain. A single computer model could not represent both the behaviour and structure of the brain simultaneously.

020179.1

ON SYSTEMS WITH REDUNDANCY AND THEIR INHERENT BOUNDS ON ACHIEVABLE RELIABILITY, by Peter W. Becker, *Electronics Institute, Electronics Laboratory, Technical University of Denmark, Building* 344, *DK*-2800 *Lyngby, Denmark*, 8 pp. ($1.00).

Consider a system which is supposed to bring some commodity (e.g. electric power, gas, oil, telephone messages) from one point P to another point T. To avoid total disruption of the services several parallel connections between P and T may be built, i.e. the designer tries to enhance the system's reliability through use of redundancy. In the paper a method, the so-called Θ-transformation, is described by which the highest and lowest system reliability achievable may be determined for a redundant configuration. As a by-product we become able to pinpoint the statistical relationships which give rise to highest and lowest system reliabilities.

021679

IS THERE A GHOST IN THE MACHINE AFTER ALL? by Gordon G. Globus, *Department of Psychiatry and Human Behavior, California College of Medicine, University of California at Irvine.* 16 pp. ($2.00).

The consequences of admitting an abstract subject to general systems —a "ghost in the machine"—are informally considered in relation to Gödel's Incompleteness Theorem. This subject is represented as a singularity in analyticity. Gödel's demonstration of a self-referential formula, G_0, undecidable in a system Z, rests on the assumption that self-reference can only be to "self" as an ordinary analytical object. But when reference to "self" as subject is allowed in a system Z^*, there occurs a new formula, G_S, which is decidable in Z^* and which precludes the undecidable G_0 from Z^*. Whereas Z is "incomplete" by virtue of an undecidable formula, Z^* is "complete" but contains a singularity representing subjectivity. The relative advantages of Z and Z^* are considered. Admission of a "ghost" to the machine provides a novel area of exploration for general systems theorists.

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REPORT ON THE DECIPHERMENT OF THE AMERICAN STRIP CIPHER O-2 BY THE GERMAN FOREIFN OFFICE (1945), Hans Rohrbach.

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SECURITY OF NUMBER THEORETIC PUBLIC-KEY CRYPTOSYSTEMS AGAINST RANDOM ATTACK, Bob Blakely, G.R. Blakely.

CASANOVA AND THE BEAUFORT CIPHER, Brian J. Winkel.

COURSES IN CRYPTOLOGY.

Subscription $16 (US) per year. CRYPTOLOGIA, Albion College, Albion, MI 49224 USA.
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Calendar

August 6–17, 1979

FIRST INTERNATIONAL SUMMERSCHOOL ON SYSTEMS METHODOLOGY
Delft, The Netherlands. *Contact* Dr. G. Broekstra, Graduate School of Management, Poortweg 6–8, Delft, The Netherlands.

August 20–24, 1979

SIXTH INTERNATIONAL JOINT CONFERENCE ON ARTIFICIAL INTELLIGENCE
Tokyo, Japan. *Contact* Raj Reddy, Department of Computer Science, Carnegie-Mellon University, Schenley Park, Pittsburgh, Pennsylvania 15213, U.S.A.

August 21–24, 1979

INTERNATIONAL MEETING OF THE SOCIETY FOR GENERAL SYSTEMS RESEARCH
London, England. *Contact* Richard Ericson, George Washington University, Lisner Hall, 2023 G. Street N.W., Washington D.C. 20006, U.S.A.

August 22–29, 1979

SIXTH INTERNATIONAL CONGRESS OF LOGIC, METHODOLOGY AND PHILOSOPHY OF SCIENCE
Hanover, Federal Republic of Germany. *Contact* Secretariat des Internationalen Wissenschatten Welfengarten 1, D-3000 Hanover, BRD.

September 10–12, 1979

SEVENTH INTERNATIONAL CONGRESS OF MATHEMATICAL BIOLOGY
Paris, France. Theme of the Congress: Mathematical Theory of Structure; Application to Sociological, Biological, Chemical and Physical Systems. *Contact* Société Internationale de Biologie Mathématique, 11 bis, Avenue de la Providence, 92160 Antony, France.

September 10–14, 1979

NINTH INTERNATIONAL CYBERNETICS CONGRESS
Namur, Belgium. *Contact* J. Lemaire, Manogring Directos, IAC, Palais des Expositions, Place André Rijckmans, B-5000, Namur, Belgium.

October 8–12, 1979

INTERNATIONAL SYMPOSIUM ON SYSTEM-MODELING-CONTROL
Zakopane, Poland. *Contact* Center of Electronic Computing, Technical University of Lodž, UL. Piotrkowska 220, 90–369 Lodž, Poland.

January 3–4, 1980

THE 13TH. HAWAII INTERNATIONAL CONFERENCE ON SYSTEMS SCIENCES
Ilikai Hotel, Honolulu, Hawaii. *Contact* Ralph H. Sprague, Jr., College of Business Administration, University of Hawaii, 2404 Maile Way, Honolulu, Hawaii 96822, U.S.A.

January 7–10, 1980

TWENTY-FOURTH ANNUAL MEETING OF THE SOCIETY FOR GENERAL SYSTEMS RESEARCH
Holiday Inn, 480 Sutter St., San Francisco, California 94108, U.S.A. Program Theme: "Systems Science and Science." *Contact* Bela H. Banathy, Far West Laboratory for Educational Res. and Dev., 1855 Folsom Street, San Francisco, California 94103, U.S.A. Telephone: (415)-565-3197.

April 8–11, 1980

FIFTH EUROPEAN MEETING ON CYBERNETICS AND SYSTEMS RESEARCH
Vienna, Austria. *Contact* Robert Trappl, Österreichische Studiengesellshaft für Kybernetik, Schottengasse 3, A-1010 Wien 1, Austria. The conference will include a Symposium on "General Systems Methodology." Papers submitted to this Symposium should be sent to George J. Klir, Dept. of Systems Science, SAT, SUNY, Binghamton, New York 13901, U.S.A.

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FUZZY

sets and systems

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Aims and Scope:

The primary purpose of the INTERNATIONAL JOURNAL FOR FUZZY SETS AND SYSTEMS is to serve as a means of *professional communication between scientists and practitioners* being interested, doing research or applying Fuzzy Sets.

Fuzzy Sets have become a concern and a tool of persons working in very different areas: mathematicians in many different areas such as logic, topology, algebra, etc. are doing research in the area of Fuzzy Sets. The same is being done in the area of general system (intelligent systems) of decision theory and optimization, of approximate reasoning and sociology. Moreover, the theory of Fuzzy Sets is already applied in a variety of areas such as control theory, medicine, information processing, cluster analysis, genetics, electrical engineering etc..

Most of the contributions to the theory of Fuzzy Sets and of their applications are distributed over a very wide range of scientific journals, thus the transparency of knowledge in the area of Fuzzy Sets nowadays is very small, and scientists working in the area of fuzzy sets in different disciplines or on different continents do not know of each other and of their contributions.

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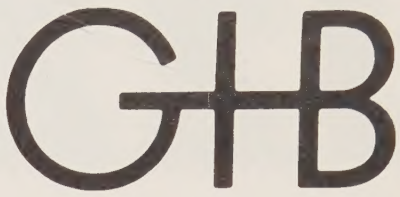

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